D.C. Track Circuit Measurements

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Method of determining rail and ballast resistance, which, although based on exact mathematics, is simpler to apply than the standard closed-circuit method

THE usual methods used in determining rail and ballast resistance are faulty, both from the practical and theoretical standpoints. Since the formulae are based on taking simultaneous voltage and current readings at each end of a circuit with the relay in service, there is always a greater chance for error than if readings were taken only at the feed end. This is true especially of a circuit fed directly from a rectifier on a fluctuating a.c. line. The formulae are faulty theoretically, since in measuring rail resistance the true effect of ballast is ignored and, vice versa, in determining ballast resistance, the effect of rail resistance is not properly considered. In a good circuit these discrepancies have little influence and it is possible to take fairly accurate measurements by accepted methods. However, it is usually a comparatively long and poor circuit which gives trouble enough to warrant measuring. Such a circuit is apt to be one in which the total rail resistance is greater than the total ballast resistance. When measuring a circuit of this nature by any of the usual methods, rail resistance as measured is lower than the true value and ballast resistance measures too high.

Track Circuit Mathematics

Assuming uniform distribution, it is possible to accurately determine rail and ballast resistance by employing pure track circuit mathematics. An added advantage of such a method is that all meter readings can be taken from the feed end. It can be proved that in a conventional end-fed d.c. track circuit with uniform distribution of rail and ballast resistance:

$$E_{TF} = E_{TR}\cosh\sqrt{\frac{R}{B}} + I_R\sqrt{RB}\,\sinh\sqrt{\frac{R}{B}} \qquad (1)$$

$$I_F = I_R\cosh\sqrt{\frac{R}{B}} + \frac{E_{TR}}{\sqrt{RB}}\sinh\sqrt{\frac{R}{B}} \qquad (2)$$

where:

E_{TF} = track voltage, feed end
E_{TR} = track voltage, relay end
I_F = current, feed end
I_R = current, relay end
R = total rail resistance of circuit
B = total ballast resistance of circuit
cosh = hyperbolic cosine
sinh = hyperbolic sine

To employ these equations in track circuit measurements, two sets of readings should be taken. One set is taken with a shunt of negligible resistance across the rails at the relay end. Using equations (1) and (2) in this special case, the track voltage at the relay end is zero, then:

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$$E_{TF} = I_R\sqrt{RB}\,\sinh\sqrt{\frac{R}{B}}$$

$$I_F = I_R\cosh\sqrt{\frac{R}{B}}$$

Under this condition E_{TF} divided by I_F is an approximation of the rail resistance and shall be identified as R_M, the metered rail resistance:

$$R_M = \frac{I_R\sqrt{RB}\,\sinh\sqrt{\frac{R}{B}}}{I_R\cosh\sqrt{\frac{R}{B}}} = \sqrt{RB}\tanh\sqrt{\frac{R}{B}} \qquad (3)$$

The second set of readings must be taken with the circuit open, i.e., without a shunt and with the relay disconnected from the track circuit. In this case the current at the relay end, I_R, is zero. Again using equations (1) and (2):

$$E_{TF} = E_{TR}\cosh\sqrt{\frac{R}{B}}$$

$$I_F = \frac{E_{TR}}{\sqrt{RB}}\sinh\sqrt{\frac{R}{B}}$$

Under this special condition E_{TF} divided by I_F is an approximation of the ballast resistance and may be identified as B_M, the metered ballast resistance:

$$B_M = \frac{E_{TR}\cosh\sqrt{\frac{R}{B}}}{\frac{E_{TR}}{\sqrt{RB}}\sinh\sqrt{\frac{R}{B}}} = \frac{\sqrt{RB}}{\tanh\sqrt{\frac{R}{B}}} \qquad (4)$$

Dividing (3) by (4):

$$\frac{R_M}{B_M} = \frac{\sqrt{RB}\tanh\sqrt{\frac{R}{B}}}{\frac{\sqrt{RB}}{\tanh\sqrt{\frac{R}{B}}}} = \tanh^2\sqrt{\frac{R}{B}} \qquad (5)$$

Extracting the square root:

$$\sqrt{\frac{R_M}{B_M}} = \tanh\sqrt{\frac{R}{B}} \qquad (6)$$

Multiplying both sides of equations (3) and (4) together, we obtain the fundamental relation:

$$R_M B_M = RB \qquad (7)$$

Multiplying both sides of equation (7) by R/B:

$$\frac{R}{B}R_M B_M = R^2$$

$$R = \sqrt{\frac{R}{B}}\,\sqrt{R_M B_M} \qquad (8)$$

Rearranging (5):

$$B_M = \frac{R_M}{\tanh^2 \sqrt{\frac{R}{B}}}$$

Substituting this value of B_M into (8):

$$R = \sqrt{\frac{R}{B}}\sqrt{R_M \frac{R_M}{\tanh^2 \sqrt{\frac{R}{B}}}} = \sqrt{\frac{R}{B}}\sqrt{\frac{R_M^2}{\tanh^2 \sqrt{\frac{R}{B}}}}$$

$$= \sqrt{\frac{R}{B}}\,\frac{R_M}{\tanh \sqrt{\frac{R}{B}}} = \frac{\sqrt{\frac{R}{B}}}{\tanh \sqrt{\frac{R}{B}}}\,R_M \qquad (9)$$

Substituting this value of R into equation (7):

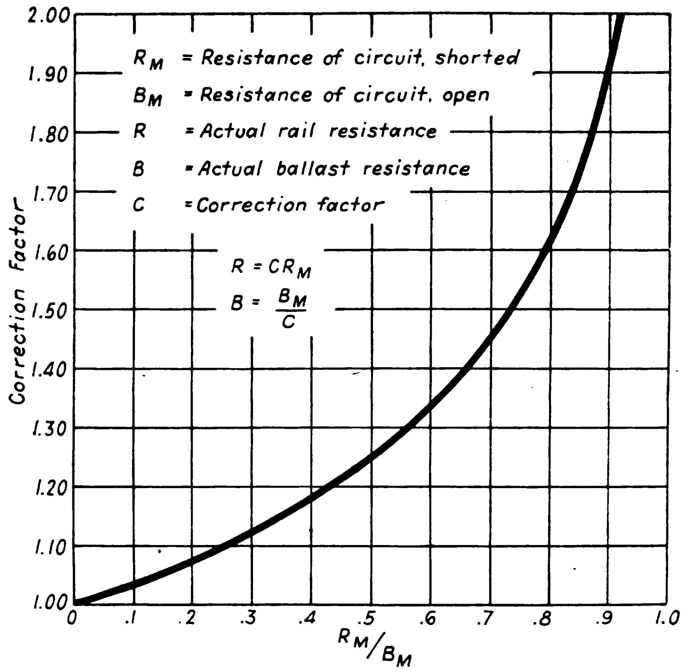
$$R_M B_M = \frac{\sqrt{\frac{R}{B}}}{\tanh \sqrt{\frac{R}{B}}}\,R_M B$$

$$B_M = \frac{\sqrt{\frac{R}{B}}}{\tanh \sqrt{\frac{R}{B}}}\,B$$

$$B = \frac{B_M}{\dfrac{\sqrt{\frac{R}{B}}}{\tanh \sqrt{\frac{R}{B}}}} \qquad (10)$$

Since R_M and B_M are known values, computed from meter readings, we have a means of determining R and B, the unknown factors of the track circuit. The $\tanh \sqrt{\frac{R}{B}}$ is first computed by means of equation (6). Then, using a table of hyperbolic functions, $\sqrt{\frac{R}{B}}$ itself is determined. The quotient of $\sqrt{\frac{R}{B}}$ divided by $\tanh \sqrt{\frac{R}{B}}$ is a correction factor which is used to convert R_M and B_M to R and B respectively. As will be noted in equations (9) and (10), multiplying R_M by the correction factor, we obtain R, and dividing B_M by the same factor, we obtain B.

Fortunately, it is possible to forego the greater part of the mathematics involved and still use this method of track circuit measurement. By assuming values of R_M/B_M in uniform steps and determining the corresponding correction factors, a curve may be plotted from which the correct correction factor may be determined for any value of R_M/B_M. Such a curve is illustrated here and for the convenience of those who may wish to reproduce it on a larger scale, an abbreviated table of correction factors is shown. As supplementary information, the corresponding R/B values are also given. In an attempt to make



Graph, showing the curve from which the correct correction factor may be determined for any value of ratio R_M/B_M

$\frac{R_M}{B_M}$	Correction Factor	$\frac{R}{B}$
.00	1.0000	.0000
.05	1.0172	.0517
.10	1.0355	.1072
.15	1.0550	.1670
.20	1.0760	.2316
.25	1.0986	.3017
.30	1.1230	.3784
.35	1.1496	.4626
.40	1.1787	.5558
.45	1.2108	.6597
.50	1.2464	.7768
.55	1.2864	.9102
.5800	1.3130	1.0000
.60	1.3319	1.0644
.65	1.3844	1.2458
.70	1.4461	1.4639
.75	1.5207	1.7344
.80	1.6140	2.0841
.85	1.7375	2.5661
.88	1.8355	2.9647
.90	1.9168	3.3067
.9168	2.0000	3.6672

Table of correction factors.

the curve applicable to the poorest track circuit which may be encountered, it has been extended to what may appear to be a ridiculously high R_M/B_M ratio.

For the benefit of those who may have omitted reading the detailed mathematical explanation of this method of track circuit measurement, it may be briefly summarized: The track circuit is first measured with a very low resistance shunt clamped across the rails at the relay end. Commercially-made shunt connectors are available for this purpose. The resistance of the circuit is computed from the voltage and current readings taken at the feed end. Since this value is influenced mostly by the rail resistance, it is called R_M. The circuit is then measured again from the feed end without any load other than the natural ballast. The resistance computed from readings taken under this condition is called B_M, since it is influenced mostly by the ballast. Divide R_M by B_M. From the correction curve, find the correction factor corresponding to this R_M/B_M ratio. Multiply R_M by the correction factor to get the actual rail resistance. Divide B_M by the correction factor to get the ballast resistance.

The correction factor method may be used in measuring center-fed circuits, provided that the feed is reasonably near the center. In determining R_M the circuit must be shorted at both ends and in measuring B_M, it must be open at both ends. The same correction curve may be used in determining R and B. However, if the correction factor is designated as C:

$$R = 4CR_M$$

but:

$$B = \frac{B_M}{C}$$

The R/B ratios given in the tables for end-fed circuits must also be multiplied by 4 to be applicable to center-fed circuits. The advantage of this method of measurement is that all readings are taken from one point. Assuming uniform distribution, the accuracy is not affected by the quality of the track circuit. Although based upon rather complicated mathematics, there are fewer mathematical steps involved in this method of computation than in the standard closed circuit method and each of these may be worked out on an ordinary slide rule.