

# Mathematics of the A-C. Track Circuit

*A study of the fundamental characteristics of a-c. track circuits for the benefit of those responsible for reliable operation*

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IN order to understand the operation and proper adjustment of the two-element induction type of a-c. relay, and be able to calculate the phase relations, it is necessary to have some knowledge of trigonometric functions, particularly those which apply to the solution of the right angle triangle. The three functions we especially need to remember are: Sin  $A$  equals  $\frac{a}{c}$ . Cos  $A$  equals  $\frac{b}{c}$ . Tan  $A$  equals  $\frac{a}{b}$ . Figure 1 shows how these ratios are obtained.

When the rotating vector  $OP$ , which forms the hypotenuse of the triangle  $ABC$ , is in the positions shown, it makes a certain projection on both the  $yy$  and  $xx$  axes. That is, a line drawn parallel to the  $X$  axis from  $P$ , gives the projection of  $OP$  on the  $Y$  axis. A line drawn parallel to the  $Y$  axis and downward from  $P$ , gives the projection of  $OP$  on the  $X$  axis. The ratio of this projection on the  $Y$  axis to the rotating vector  $OP$  gives the sine of the angle that  $OP$  makes with the  $X$

axis. Therefore, Sin  $A$  equals  $\frac{BC}{OP}$  or  $\frac{a}{c}$ , and when angle  $A$  is 30 deg. this ratio is 0.5. Similarly the ratio of the projection of  $OP$  on the  $X$  axis to  $OP$ , is called the cosine of angle  $A$ , and when angle  $A$  is 30 deg. this

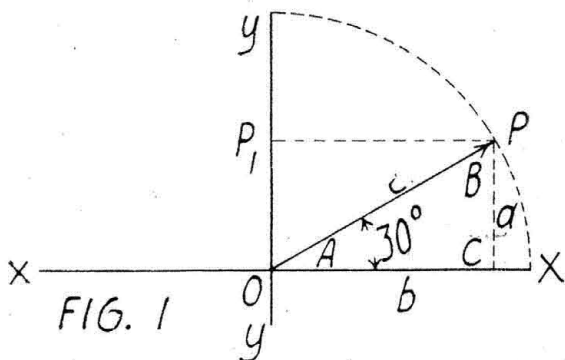


FIG. 1—Graphic explanation of trigonometric functions

is 0.866. The ratio of  $BC$  to  $AC$  gives the tangent of angle  $A$  and for a 30 deg. angle the ratio of these two sides is always 0.577.

The ratio of the sine to the cosine also gives the tangent, i.e.,  $\frac{0.5}{0.866}$  equals 0.577. As the rotating vector  $OP$

is rotated from zero to 90 deg. it is evident that the sine will vary from zero to one, and the cosine from one to zero. Since the sine divided by the cosine gives the tangent, it is also evident that this must vary from zero to infinity. These ratios remain the same for any given angle regardless of the length of the sides, and tables

\*Mr. Lofstrand has had several years' experience in the installation and maintenance of a-c. signaling, and after pursuing a course at the University of Wisconsin, secured the aid of the university's electrical engineering department in solving some of his problems in the field. The relays referred to are the General Railway Signal Company's type and engineers of that company have checked this article.—Editor.

have been prepared giving their value for each degree of angle. The equations given above can be solved algebraically. That is, knowing any two quantities one can easily solve for the other.

## Applying Method to A-C. Track Circuits

These relations of the sides and angles of a right-triangle to each other will now be applied to the solution of a-c. track circuit problems. The effect of self-induction is to cause the current to lag (in respect to time) behind the pressure. (It is not the purpose of this article to go into detail and show how and why this is so, as this subject is covered in text books on electricity.) If it were possible to design a circuit so that it had no resistance, this time lag would be 90 deg. which means that at the instant the pressure reaches

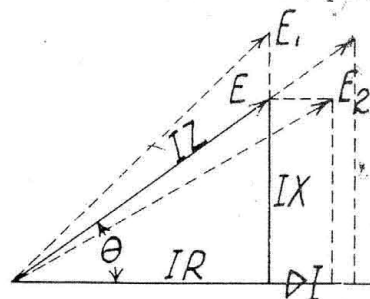


FIG. 2

Fig. 2—Vector diagram showing relations of elements in a circuit

its maximum value, the value of the current is zero. It does not mean that the current flows at a right angle to the pressure, but, owing to the fact that the current and pressure are varying from zero to maximum in accordance with the sine law,\* it can be expressed in degrees. By phase difference is meant the time interval (expressed angularly) between the maximum values, and the vector diagram is merely a means of representing this time interval as an angular function. If we have a circuit that has resistance only, the pressure and current will be in phase.

Evidently then, if the voltage across a reactance is 90 deg. ahead of the current, and the voltage across a resistor in series thereto is in phase with the current, the two voltages are at right angles to each other, and the resultant voltage (series) will be their vector sum. For this reason any voltage reading taken across a device having both resistance and inductance is really the resultant of two voltages at right angles to each other, in a vector sense. The component in phase with the current is  $IR$ , (see Fig. 2) and is called the "in phase" or cosine component. The component 90 deg. ahead of the current is  $IX$  and is called the "wattless" or sine component. The resultant is  $IZ$ , or the hypotenuse of the triangle. Figure 2 is a vector diagram of these relations.

\*Refer to any text book on a-c. theory for explanation of sine wave form of voltage and current.

not change the power factor of the local winding, but rather the local circuit. That is, the winding itself will still have the same power factor but the current will be more nearly in phase with the impressed voltage.

The new angle will be  $\theta = \tan^{-1} \left( \frac{X}{R+r} \right)$  where (r) is

the added resistance. There is seldom any difficulty in getting sufficient phase displacement on long track circuits, and no doubt there are times when the ballast resistance gets so low that the angle is near the ideal of 90 deg. However, the voltage would probably be below the pick-up value in this case, unless the voltage in dry weather was kept rather high, but fortunately, the improved phase conditions compensate to a great extent for the loss in voltage, as the ballast resistance goes down. Unfortunately, this last statement also holds good for a shunt on the track circuit, as a poor train shunt is equivalent to low ballast resistance and produces ideal phase conditions.

The calculations for phase conditions in this article refer only to single phase connections. Where three phases are available it is possible to get almost any phase displacement desired.

## Radio Applications on the Railroads\*

By O. H. Caldwell

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**A**LREADY radio contributes to the operation of the modern American railroad in many ways. For example, radio telephone communication is today being operated experimentally between caboose and locomotive on freight trains. At either end of the train, a member of the train crew can pick up a transmitter, talk into it and his voice will be heard with loud-speaker volume at the far end, a mile away, above all the noise of the train.

Similarly, radio telephony is being used at gravity switching yards for communicating between switch towers and locomotives. In this way time is saved, costs are reduced, and the safety of the men is increased. The possibility of keeping in continuous communication with freight trains from towers along the right-of-way is another development that is already in sight. The Federal Radio Commission has set aside five of the valuable short wave length channels for the use of transmitters operated by railroads, either on railroad rolling stock or on railroad tug boats in harbors. The railroad channels are designated as 2,410, 2,422, 2,440, 2,450 and 2,470 kilocycles.

Radio also presents possibilities for the greater convenience and comfort of the passengers. In Germany and Austria, for instance, passengers on fast trains between Berlin and Hamburg can maintain telephone conversations with their offices or homes in either city while they speed along at 60 m.p.h. Yet here in America, with our lavish scale of expenditure on traveling comfort, we are told that such telephone service would not pay. On some of the fast Canadian trains and on certain trains in the United States, each chair in the club car is equipped with head sets for listening to broadcasting.

Consider the dreary isolation of the average railroad

waiting room, which might well be equipped with a loud speaker operated not by radio but over the telegraph lines of the railroad without disturbing in any way the message traffic on those wires. This would be wired broadcasting, not wireless. The railroad would be able to control its own program, and along with musical features, could interpolate valuable public-relations messages, information for shippers and the public. In this way any railroad could build up at small cost and a little effort of a few men, a chain of some hundreds or thousands of outlets which would render great service to its neighbors and patrons. Already the new owners of one railroad in Illinois have used radio broadcasting to educate the farmers along the line, thus improving their production and prosperity, and so increasing and stabilizing the traffic.

Broadcasting stations themselves have been employed for dispatching trains during emergencies, such as great sleet storms and other general wire interruptions. The 600 broadcasting stations which are now licensed, although only 160 can operate simultaneously, provide a great network available for emergency communication when all other means fail.

Therefore, radio is rendering many valuable services to the railroads of this country, and as time goes on, I believe, it is destined to serve on even a greater scale. But as you railroad men make future demands on radio service, I beg of you to remember that we have only this one great common conductor, which you can think of as a vast copper table. We must impose all of these thousands of services by using different frequencies. Therefore, I believe that your guiding policy should be to require no service of the radio where wires can perform equally as well. Where a wire circuit can be used for communication purposes, I believe that engineering opinion will show that the wire is always more dependable, more efficient and more economical than a corresponding radio circuit for communication purposes. Since our radio waves are so few, I hope you will help us to preserve them for the essential humanitarian services of life-saving, both at sea and in the air, where only radio can do the job.



Recent semaphore signaling on the Coast Lines of the Atchison, Topeka & Santa Fe

\*Abstracted from address made at the A. R. E. A. annual convention on March 7, 1929, at Chicago. Mr. Caldwell was formerly a member of the Federal Radio Commission.