Some Hints on Track Circuit Calculation.

By Ture Hård,
Chief Engineer for Signals, Swedish State Railway, Stockholm.

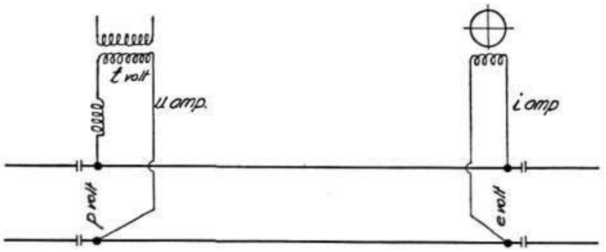
The solving of track circuit problems by considering the circuit as a series-parallel circuit with uniformly distributed constants is not a new idea. The method of calculation described by Harold Mc Ready in his book "Alternating Current Signalling" is based on this principle and is now generally used for practical purposes.

The method presented below is also based on this same principle but differs essentially from the former method with respect to the practical treatment of the problem.

The newer method is practicable for both direct and alternating currents and allows a mathematical treatment to be used throughout. The comparison of different track conditions and apparatus is easier, and the effect of shunting can be more readily calculated. A method of calculating the rail impedance and the ballast resistance from practical tests on track circuits can be based on the same fundamental principles and is described in the following.

General assumptions.

A track circuit, the general arrangements of which are shown in fig. 1, may be considered as a power



R 878 Fig. 1.

distribution system in which the line wires are the rails which transmit power from the track transformer or the battery to the track relay.

The leakage resistance of the ballast is considered equally distributed over the whole length of the track.

The capacity between the two rails is considered negligible as compared with the ohmic leakage. At every point of the track, therefore, the current through the ballast is in phase with the voltage across the rails at that point.

The self inductance of the rail must be taken into account by using values obtained from actual measurements on track circuits.

In the equations and formulæ presented below the following notations will be used. A letter in bold-faced type signifies a vector or complex quantity. When such a letter is met with in an equation or formula, the phase angle must be taken into consideration. If regular type is used, this means that only the magnitude of the vector or quantity is to be considered. Phase angles are denoted by capital letters.

z = Rail impedance in ohms per 1000 feet of track.
r = Ballast resistance in ohms per 1000 feet of track.
Z = Phase angle of the rail impedance z.
l = Length of track circuit in thousands of feet.
$\mathbf{e}$ = Volts between rails at relay end.
$\mathbf{i}$ = Ampères in rails at relay end.
$\mathbf{p}$ = Volts between rails at feed end.
$\mathbf{u}$ = Ampères in rails at feed end.
Fe, Fp and Fu = Phase angles of $\mathbf{e}$, $\mathbf{p}$ and $\mathbf{u}$ with respect to $\mathbf{i}$, which is taken as the axis of reference.

In the equations and diagrams below, the angles are reckoned positive in the counter-clockwise direction. The phase angle of an impedance is considered positive when the current lags after the voltage.

General equations and formulæ.

With the foregoing assumptions the following equations can be shown to represent the conditions prevailing in the track circuit

$$\mathbf{u} = \left(\mathbf{i} + \mathbf{e}\frac{\mathbf{b}}{\mathbf{a}}\right)\mathbf{c} \qquad (1)$$

$$\mathbf{p} = (\mathbf{e} + \mathbf{i}\ \mathbf{a}\ \mathbf{b})\ \mathbf{c} \qquad (2)$$

where

$\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are complex quantities, whose magnitudes a, b and c, and phase angles A, B and C, can be calculated from the following formulæ

$$a = \sqrt{rz} \quad \text{(3)}$$

$$A = \frac{Z}{2} \quad \text{(4)}$$

$$b = \sqrt{\frac{\cosh 2m - \cos 2n}{\cosh 2m + \cos 2n}} \quad \text{(5)}$$

$$B = \text{arctg } \frac{\sin 2n}{\sinh 2m} \quad \text{(6)}$$

$$c = \frac{1}{\sqrt{2}}\sqrt{\cosh 2m + \cos 2n} \quad \text{(7)}$$

$$C = \text{arctg } (\text{tgh } m \times \text{tg } n) \quad \text{(8)}$$

where

$$m = l\sqrt{\frac{z}{r}} \cos \frac{Z}{2} \text{ and } n = l\sqrt{\frac{z}{r}} \sin \frac{Z}{2}.$$

From the formula (3) it is clear that the value of a is independent of the length of the track circuit, and for a given track proportional to the square root of the ballast resistance.

Further, from formula (4) we find that for a given track the phase angle A is constant and equal to the half of the phase angle Z of the rail impedance.

From the formulæ (5), (6), (7) and (8), lastly, we find that b, B, c and C are functions of the quantity $l\sqrt{\frac{z}{r}}$ and the phase angle Z. With the values of l, r, z and Z given, the values of b, B, c and C can be calculated with the aid of tables for trigonometric and hyperbolic functions available in most engineering handbooks.

To illustrate the variations of the quantities b, B, c and C, the curves in fig. 2 have been plotted in the following manner.

The values of b, B, c and C have been calculated for a series of values of $l\sqrt{\frac{z}{r}}$ and Z. For each angle Z curves representing b, B, c and C have then been plotted by using the values of $l\sqrt{\frac{z}{r}}$ as abscissæ and the corresponding values of b, B, c and C as ordinates.

The curves for b and c for $Z = 0$ apply to direct current. With $Z = 0$ the value of B and C is zero.

If plotted with sufficiant accuracy, the curves can be used to determine the values of b, B, c and C for

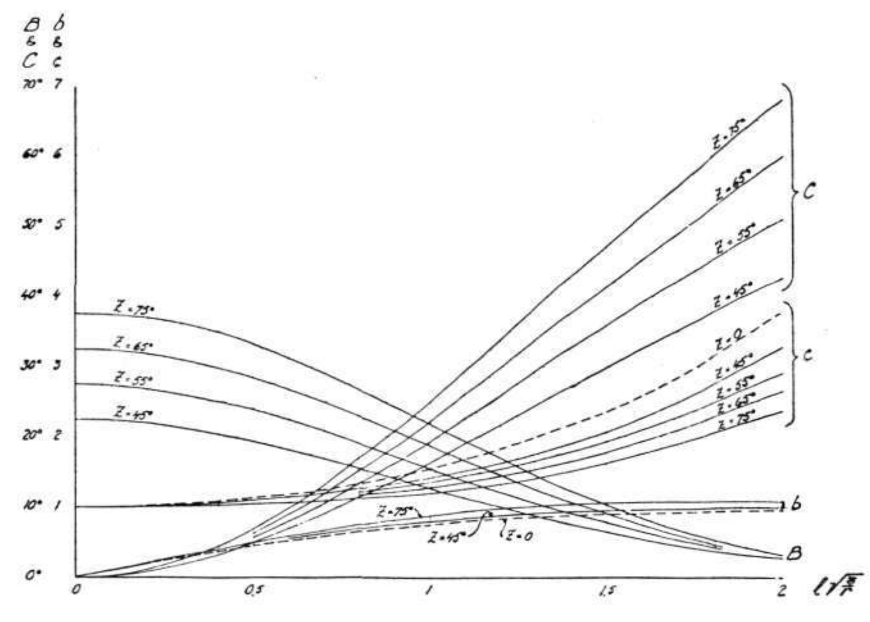


Fig. 2.

any value of $l\sqrt{\frac{z}{r}}$ and Z by seeking the ordinates corresponding to the given abscissa $l\sqrt{\frac{z}{r}}$ and phase angle Z. When the given angle Z lies between two of the phase angle values for which curves have been plotted, the proper ordinates b, B, c and C may be determined by means of interpolation.

Example 1.

Track circuit, 5000 feet long.

Ballast resistance, 6 ohms per 1000 feet.

Rail impedance, .31 ohms per 1000 feet of track at $\cos Z = .68$.

Hence $l = 5$; $r = 6$; $z = .31$; $Z = 47°$.

Then $l\sqrt{\frac{z}{r}} = 1.14$; $a = \sqrt{rz} = 1.36$; $A = \frac{Z}{2} = 23.5°$.

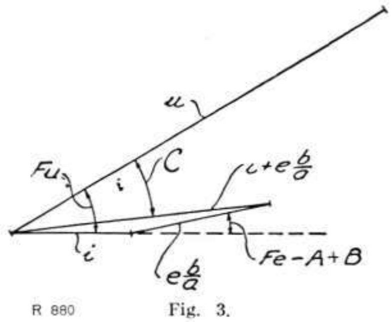
From the formulæ (5), (6), (7) and (8) or from the curves in fig. 2, we obtain

$b = .86$; $B = 11.5°$; $c = 1.53$; $C = 21°$.

Calculation of the current and voltage at the feed end.

When the magnitudes of a, b and c, as well as the phase angles A, B and C, are known, it is possible from the equations (1) and (2) to determine the voltage and current to be used at the feed end of the track circuit in order to maintain a given current and voltage at the relay end.

The determination may be accomplished either graphically by plotting vector diagrams, or analytically by means of vector algebra.



R 880 Fig. 3.

The graphical solution of equation (1)

$$\boldsymbol{u} = \left(\boldsymbol{i} + \boldsymbol{e}\frac{\boldsymbol{b}}{\boldsymbol{a}}\right)\boldsymbol{c}$$

is shown in fig. 3.

The current i is laid off along the axis of reference ($Fi = 0$). The vector representing the expression $\boldsymbol{i} + \boldsymbol{e}\frac{\boldsymbol{b}}{\boldsymbol{a}}$ is then found by adding a vector of the length $e\frac{b}{a}$ to the current vector i so as to form an angle $Fe - A + B$ with the axis of reference.

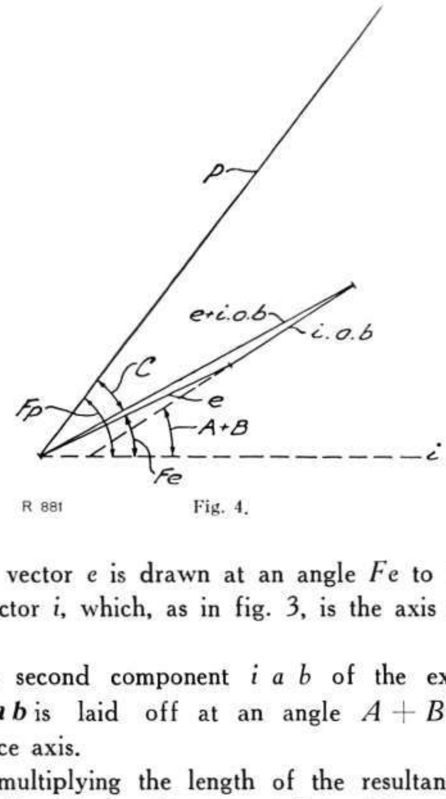
The magnitude of the current vector u is obtained by multiplying the length of $\boldsymbol{i} + \boldsymbol{e}\frac{\boldsymbol{b}}{\boldsymbol{a}}$ with the factor c.

The phase angle Fu of the vector u is determined by revolving the vector $\boldsymbol{i} + \boldsymbol{e}\frac{\boldsymbol{b}}{\boldsymbol{a}}$ the angle C.

The graphical solution of equation (2)

$$\boldsymbol{p} = (\boldsymbol{e} + \boldsymbol{i}\ \boldsymbol{a}\ \boldsymbol{b})\ \boldsymbol{c}$$

is shown in fig. 4.



R 881 Fig. 4.

The vector e is drawn at an angle Fe to the current vector i, which, as in fig. 3, is the axis of reference.

The second component $i\ a\ b$ of the expression $\boldsymbol{e} + \boldsymbol{i\,a\,b}$ is laid off at an angle $A + B$ to the reference axis.

By multiplying the length of the resultant vector $e + i\,a\,b$ with the factor c, and revolving this vector the angle C, the vector p is determined as to magnitude and phase.

The magnitudes p and u and their phase angles Fp and Fu may also be determined by solving the vector equations (1) and (2) analytically. For this purpose the equations should be written in the following forms where the letter j signifies the imaginary component of a complex quantity and E the base of the Naperian logarithms.

$$u\,E^{jFu} = \left[i + e\frac{b}{a}\,E^{j(Fe-A+B)}\right] c\;E^{jC} =$$

$$= \Big[i + e\frac{b}{a}\cos(Fe - A + B) +$$

$$+ je\frac{b}{a}\sin(Fe - A + B)\Big]\,c\,E^{jC}.$$

$$p\,E^{jFp} = [e\;E^{jFe} + i\;a\;b\;E^{j(A+B)}]\;c\;E^{jC} =$$

$$= [e + i\;a\;b\;E^{j(A+B-Fe)}]\;c\;E^{j(C+Fe)} =$$

$$= [e + i\;a\;b\;\cos(A + B - Fe) +$$

$$+ j\;i\;a\;b\;\sin(A + B - Fe)]\;c\;E^{j(C+Fe)}.$$

Hence

$$u = c\sqrt{\left[i + e\frac{b}{a}\cos(Fe - A + B)\right]^2 + \left[e\frac{b}{a}\sin(Fe - A + B)\right]^2} =$$

$$= c\sqrt{i^2 + \left(e\frac{b}{a}\right)^2 + 2\,i\,e\frac{b}{a}\cos(Fe - A + B)} \quad \ldots(9)$$

$$p = c\sqrt{[e + i\;a\;b\;\cos(A + B - Fe)]^2 + [i\,a\,b\sin(A + B - Fe)]^2} =$$

$$= c\sqrt{e^2 + (i\,a\,b)^2 + 2\;i\;e\;a\;b\;\cos(A + B - Fe)} \quad (10)$$

$$Fu = \text{arctg}\frac{e\dfrac{b}{a}\sin(Fe - A + B)}{i + e\dfrac{b}{a}\cos(Fe - A + B)} + C \quad \ldots\ldots\ldots(11)$$

$$Fp = \text{arctg}\frac{i\,a\,b\sin(A + B - Fe)}{e + i\,a\,b\cos(A + B - Fe)} + C + Fe \quad \ldots(12)$$

Example 2.

Let us use the track circuit described in example 1, and assume the current through the relay to be 1 amp. at 1.78 volts between the rails, and lagging 25° behind this voltage.

We have then $i = 1$ volt; $e = 1.78$ volts; $Fe = 25°$; $a = 1.36$; $A = 23.5°$; $b = .86$; $B = 11.5°$; $c = 1.53$; $C = 21°$; $Fe - A + B = 25° - 23.5° + 11.5 = 13°$; $A + B - Fe = 23.5 + 11.5 - 25 = 10°$.

Hence, from the formulæ (9), (10), (11) and (12)

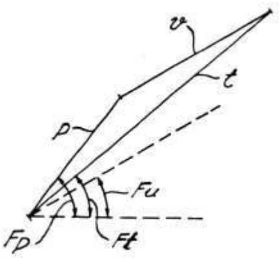
$$u = 1.53 \times \sqrt{1 + \left(1.78 \times \frac{.86}{1.36}\right)^2 + 2 \times 1 \times 1.78 \times \frac{.86}{1.36}\cos 13°} = 3.22 \text{ amp.}$$

$$p = 1.53 \times \sqrt{1.78^2 + (1 \times .86 \times 1.36)^2 + 2 \times 1 \times 1.78 \times .86 \times 1.36 \times \cos 10°} = 4.50 \text{ volts.}$$

$$Fu = \text{arctg}\frac{1.78 \times \dfrac{.86}{1.36} \times \sin 13°}{1 + 1.78 \times \dfrac{.86}{1.36} \times \cos 13°} + 21° = 28°.$$

$$Fp = \text{arctg}\frac{1 \times 1.36 \times .86 \times \sin 10°}{1.78 + 1 \times 1.36 \times .9 \times \cos 10°} + 21° + 25° = 50°.$$

If the voltage t at the track transformer secondary is given, the voltage drop v in the limiting resistance or impedance between transformer and track can be obtained as shown in fig. 5.



R 882 Fig. 5.

After plotting the vector p at an angle Fp to i, the vector v representing the voltage drop is laid off at the proper phase angle to the current u and is extended so that the resultant of p and v becomes equal to the given transformer voltage t.

Influence of a shunt between rails.

It is now easy to determine the influence on the track circuit of the application of an impedance of any sort across the rails. Let us assume an impedance of the magnitude d and with a phase angle D to be connected across the rails at the relay end.

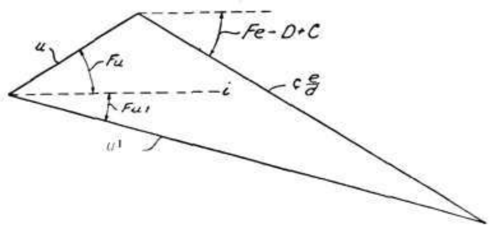
The current u_1 and voltage p_1 necessary at the feed end to maintain the voltage e at the relay end after applying the shunt d may be expressed by the following vector equations

$$u_1 = \left(i + \frac{e}{d} + e\frac{b}{a}\right)c = \left(i + e\frac{b}{a}\right)c + \frac{e}{d}c = u + \frac{e}{d}c \quad (13)$$

$$p_1 = \left[e + \left(i + \frac{e}{d}\right)a\,b\right]c = (e + i\,a\,b)\,c + \frac{e}{d}\,a\,b\,c =$$

$$= p + \frac{e}{d}\,a\,b\,c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

From the expressions (13) and (14) it follows that the current u_1 is obtained by adding to the current vector u a vector of the magnitude $\frac{e}{d}\, c$ making an angle $Fe + C - D$ with the reference axis i. The graphical construction of equation (13) is shown in fig. 6.



R 883 Fig. 6.

Likewise, the voltage p_1 can be found by adding to the voltage p a vector of the magnitude $\frac{e}{d}\, a\, b\, c$ making an angle $Fe + A + B + C - D$ with the reference axis i. The graphical solution of equation (14) is shown in fig. 7.

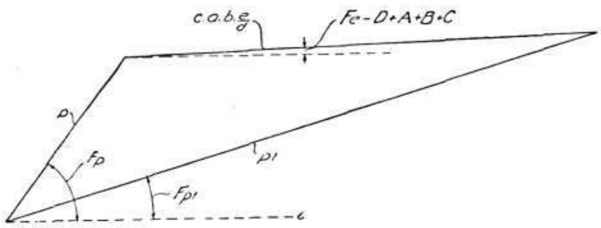


Fig 7

R 884

The solution of equations (13) and (14) may also be carried out analytically. The equations should then be written in the following forms

$$u_1 E^{jFu_1} = u\, E^{jFu} + \frac{e}{d}\, c\, E^{j(Fe+C-D)}$$

$$p_1 E^{jFp_1} = p\, E^{jFp} + \frac{e}{d}\, a\, b\, c\, E^{j(Fe+A+B+C-D)}$$

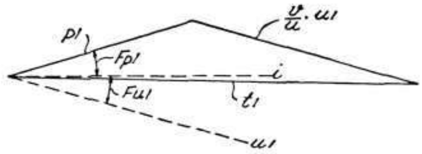
Hence

$$u_1 = \sqrt{\left[u \cos Fu + \frac{e}{d} c \cos (Fe + C - D)\right]^2 + \left[u \sin Fu + \frac{e}{d} c \sin (Fe + C - D)\right]^2} \quad \text{.........(15)}$$

$$p_1 = \sqrt{\left[p \cos Fp + \frac{e}{d} a\, b\, c \cos (Fe + A + B + C - D)\right]^2 + \left[p \sin Fp + \frac{e}{d} a\, b\, c \sin (Fe + A + B + C - D)\right]^2} \quad (16)$$

$$Fu_1 = \operatorname{arctg} \frac{u \sin Fu + \frac{e}{d} c \sin (Fe + C - D)}{u \cos Fu + \frac{e}{d} c \cos (Fe + C - D)} \quad (17)$$

$$Fp_1 = \operatorname{arctg} \frac{p \sin Fp + \frac{e}{d} a\, b\, c \sin (Fe + A + B + C - D)}{p \cos Fp + \frac{e}{d} a\, b\, c \cos (Fe + A + B + C - D)} \quad (18)$$



R 885 Fig. 8.

In fig. 8 is shown the graphical determination of the hypothetical transformer voltage t_1 corresponding to the voltage p_1, the current u_1 and the given limiting impedance $\frac{v}{u}$ between transformer and track.

After plotting the vector p_1 the voltage drop $\frac{v}{u}\, u_1$ in the limiting impedance is laid off at the proper angle to the current u_1. The resultant of p_1 and the drop vector represents the hyothetical voltage t_1.

If u, p and t are values actually existing with the track clear, it is obvious that p_1, u_1 and t_1 cannot be the real values to be expected on applying the shunt d. Since the voltage t at the track transformer secondary remains practically unchanged after the application of the shunt, it is obvious that the voltages and currents actually existing in the track circuit with the impedance d connected across the rails will be found by reducing the values p_1, u_1, e and i in the proportion of $\frac{t}{t_1}$.

Calculation of the rail impedance and the ballast resistance from "short circuit" and "open circuit" values.

Let us consider a case where the relay end is short circuited, so that the voltage between the rails is zero. From equations (1) and (2) we obtain

$$u_s = i_s c$$

$$p_s = i_s\, a\, b\, c$$

where u_s and p_s are the current and the voltage at the feed end and i_s is the current at the relay end when this latter is short circuited. Dividing the equations we obtain

$$\frac{p_s}{u_s} = a\, b \qquad (19)$$

If S is the phase angle between p_s and u_s, then $S = A + B$.

Assuming that the circuit is broken at the relay terminals so that no current passes through the relay end, then $i = 0$.

From equations (1) and (2) we then obtain

$$u_o = e_o \frac{b}{a} c$$

$$p_o = e_o\, c$$

where u_o and p_o represent current and voltage at the feed end, and e_o the voltage at the relay end when the track circuit is open at the relay end.

Dividing the equations, we obtain

$$\frac{p_o}{u_o} = \frac{a}{b} \qquad (20)$$

If O is the phase angle between p_o and u_o, then $O = A - B$.

From equations (19) and (20) the following formulæ can be derived

$$a = \sqrt{\frac{p_s\, p_o}{u_s\, u_o}} \qquad (21)$$

$$b = \sqrt{\frac{p_s\, u_o}{u_s\, p_o}} \qquad (22)$$

$$A = \frac{S + O}{2} \qquad (23)$$

$$B = \frac{S - O}{2} \qquad (24)$$

From formula (4) we get $Z = 2A$.

Hence $Z = S + O$, which means that the phase angle of the rail impedance is equal to the sum of the phase angle between current and voltage at the transformer end when the track circuit is opened at the relay end, and the phase angle between current and voltage at the feed end when the track is short circuited at the relay end.

The formulæ (5) and (6) may be written

$$\cos\, 2\, n = \frac{1 - b^2}{1 + b^2} \cosh\, 2\, m$$

$$\cosh\, 2\, m = \sqrt{1 + \cot^2 B - \cot^2 B \cos^2 2\, n}$$

By putting $\frac{1 - b^2}{1 + b^2} = k$ and solving the equations we obtain

$$\cosh\, 2\, m = \frac{1}{\cos B \sqrt{k^2 + \mathrm{tg}^2 B}} \qquad (25)$$

$$\cos\, 2\, n = \frac{k}{\cos B \sqrt{k^2 + \mathrm{tg}^2 B}} \qquad (26)$$

With the aid of the formulæ (25) and (26) the values of m and n can be calculated if the values of k and B are known.

Previously, we had

$$m = l \sqrt{\frac{z}{r}} \cos \frac{Z}{2} \quad \text{and}\; n = l \sqrt{\frac{z}{r}} \sin \frac{Z}{2}$$

Therefore,

$$\mathrm{tg} \frac{Z}{2} = \frac{n}{m}$$

$$\sqrt{\frac{z}{r}} = \frac{m}{l \cos \frac{Z}{2}} = \frac{n}{l \sin \frac{Z}{2}}$$

Further, from the general formula (3) we know that $\sqrt{r\, z} = a$.

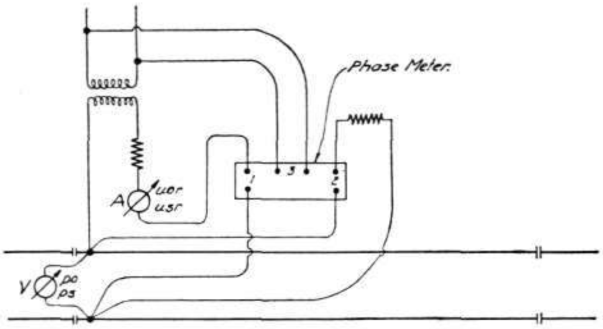
The following formulæ will, therefore, give the values of the rail impedance and the ballast resistance.

$$z = \frac{a\, m}{l \cos \frac{Z}{2}} \;\text{or}\; \frac{a\, n}{l \sin \frac{Z}{2}} \qquad (27)$$

$$r = \frac{a\, l \cos \frac{Z}{2}}{m} \;\text{or}\; \frac{a\, l \sin \frac{Z}{2}}{n} \qquad (28)$$

Method of testing.

In fig. 9 is shown a method of obtaining the values of p_o, u_o, p_s and u_s and the phase angles O and S with the aid of an ammeter, a voltmeter, and an instrument for measuring phase angles between currents.



R 886 Fig. 9.

For measuring the phase angles, a power factor meter of special design has been employed. The instrument has two separate current coils, one of which (number one) has an impedance of .6 ohms at 50 cycles and is connected in series with the ammeter.

The other current coil (number two), which has an imdepance of 16 ohms at 50 cycles, is connected across the rails in series with an ohmic resistance of about 100 ohms. To each one of the current coils belongs a tension coil (number three). Both of these tension coils are connected to the same auxiliary voltage, for instance the primary of the track transformer.

The instrument has two scales showing the phase angle between the auxiliary voltage and each of the two currents. The difference between the indicated phase angles will give the phase angle between the currents passing through the current coils. As the phase difference between the current and the total drop in the branch consisting of coil 2 and a resistance in series is known, it is also possible with the aid this instrument to determine the difference in phase between the current through the ammeter and the voltage across the rails.

In order to obtain the correct values of u_s and u_o the ammeter readings u_{sr} and u_{or} should be corrected so as to allow for current passing through the voltmeter and the phase meter coil in parallel with the voltmeter. Likewise, it will be necessary to correct the phase meter reading S_r as shown in the following example.

Example 3.

Test made on a track circuit fed with a 50 cycle alternating current.

Length of track circuit, 5000 feet.

Weight of rails, 40.5 kg. per metre.

Length of each rail, 10 metres.

Bonding by means of copper wires welded to the rail head.

Measurements with track circuit open.

Readings,

$p_o = 8$ volts (resistance of voltmeter $= 500$ ohms).

$u_{or} = 2.60$ amp.

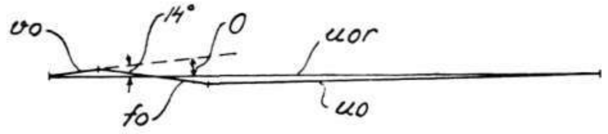
$O = 6^\circ$ (impedance of phase meter coil and resistance connected in series across the rails $=$ 110 ohms at a phase angle of 14°).

Current taken by voltmeter $v_o = \frac{8}{500} = .016$ amp.

Current taken by phasemeter $f_o = \frac{8}{110} = .073$ »

Corrected according to the diagram in fig. 10,

$u_o = 2.60 - .016 - .073 = 2.51$ amp.



R 887 Fig. 10.

Measurements with the track short circuited at relay end.

Readings,

$p_s = 3.90$ volts (resistance of voltmeter $= 100$ ohms).

$u_{sr} = 3.75$ amp.

$S_r = 54.5^\circ$ (impedance of phase meter coil and resistance $= 110$ ohms at a phase angle of 14°).

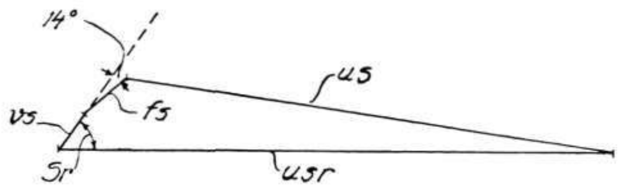
Current taken by voltmeter, $v_s = \frac{3.90}{100} = .039$ amp.

Current taken by phase meter, $f_s = \frac{3.90}{110} = .035$ amp.

Corrected according to the diagram in fig. 11.

$$u_s = 3.75 - .04 \times \cos 54.5^\circ - $$
$$- .035 \times \cos(54.5^\circ - 14^\circ) = 3.70 \text{ amp.}$$

$$S = 54.5 +$$
$$+ \frac{.04 \times \sin 54.5^\circ + .035 \times \sin 40.5^\circ}{3.70 \times 3.14} \times 180 = 55^\circ$$



R 888 Fig. 11.

Hence, from the formulæ (21), (22), (23) and (24)

$$a = \sqrt{\frac{3.90}{3.70} \times \frac{8.00}{2.51}} = 1.84$$

$$b = \sqrt{\frac{3.90}{3.70} \times \frac{2.51}{8.00}} = .575$$

$$A = \frac{55 + 6}{2} = 30.5^\circ$$

$$B = \frac{55 - 6}{2} = 24.5^\circ$$

Further, $k = \frac{1 - b^2}{1 + b^2} = \frac{1 - .33}{1.33} = .50$ and from the formulæ (25) and (26)

$$\cosh 2m = \frac{1}{.91 \times \sqrt{.25 + .21}} = 1.62$$

$$\cos 2n = .50 \times 1.62 = .81$$

$$2m = 1.06;\ m = .53.$$

$$2n = 35.45^\circ = .62;\ n = .31.$$

$$\operatorname{tg} \frac{Z}{2} = \frac{.31}{.53} = .59;\quad \frac{Z}{2} = 30.5^\circ.\quad Z = 61^\circ.$$

This value of Z coincides with the value of Z which is obtained by adding up the phase angles $S = 55^\circ$ and $O = 6^\circ$.

Finally, from the formulæ (27) and (28) we obtain

$$z = \frac{1.84 \times .53}{5 \times \cos 30.5^\circ} = .23 \text{ ohms}$$

$$r = \frac{1.84 \times 5 \times \cos 30.5^\circ}{.53} = 15 \text{ ohms.}$$

Example 4.

Track circuit, 5000 feet long, fed with direct current.

Open circuit test.

$p_o = 1.10$ volts (voltmeter resistance $= 690$ ohms).
$u_{or} = .450$ amp.

Correct $u_o = .450 - \frac{1.10}{690} = .448$ amp.

Short circuit test.

$p_s = .43$ volts (voltmeter resistance $= 690$ ohms).
$u_{sr} = 2.50$ amp.

Correct $u_s = 2.50 - \frac{.43}{690} = 2.50$ amp.

Hence,

$$a = \sqrt{\frac{.43}{2.50} \times \frac{1.10}{.448}} = .65.$$

$$b = \sqrt{\frac{.43}{2.50} \times \frac{.448}{1.10}} = .265.$$

$$k = \frac{1 - .07}{1.07} = .87.$$

Since $B = 0$, $\cos B = 1$ and $\operatorname{tg} B = 0$.

Hence, $\cosh 2\,m = \frac{1}{k} = 1.15$

$2\,m = .54;\ m = .27$

$$z = \frac{.65 \times .27}{5} = .035 \text{ ohms per 1000 feet.}$$

$$r = \frac{.65 \times 5}{.27} = 12 \text{ ohms per 1000 feet.}$$

CONTENTS: Carl Edward Nilsson †. — The telephone as an aid in the organization of the taxicab service in large cities. — The Swedish Radio Company. — The smallest electric interlocking machine. — Telephone poles of reinforced concrete. — Some hints on track circuit calculation.

Stockholm 1929. Kurt Lindberg, Boktryckeriaktiebolag.