A Method for Computing Track Circuits II

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In Ericsson Review No 2/1947 the author commenced presenting a method of computing track circuits with variable leakage. Shunt line equations were deduced for tracks shunted at the relay end or at the feed end. In the present issue the author proceeds to show how the method may be applied to track circuits with variable supply voltage. A numerical example is worked out. Shunt line equations are deduced for a modified arrangement of the track circuit and also for an arbitrary point on the track circuit.

The variation of the shunt conductance along the track during different conditions is analyzed with the aid of the shunt line equation. The possibility of building very long track circuits is also discussed. Finally is shown with the aid of the shunt line equation how to dimension the track relay.

V. The Influence of Varying Supply Voltage[1]

If the local current varies, this must be taken into consideration when the equations for the operating and release circles are deduced. Assume that one operation occurs when the local current has the value I_L and that another operation or a release occurs when the local current has the value I'_L. The ratio between the lifting forces in section I (page 38 in Ericsson Review No 2/1947) will then take the form:

$$\frac{q_C}{q_D} = \frac{k}{k_1} \cdot \frac{|I_L| \cdot |I_{RC}| \sin \varphi}{|I'_L| \cdot |I_{RC}| \sin (\varphi + \varphi_1)}$$

If we write

$$\frac{q_C}{q_D} \cdot \frac{k_1}{k} \cdot \frac{|I'_L|}{|I_L|} = f'$$

a similar release circle equation as in section I may be deduced with, however, f' exchanged for f.

As

$$f = \frac{q_C}{q_D} \cdot \frac{k_1}{k}$$

it follows that

$$f' = \left| \frac{I'_L}{I_L} \right| \cdot f$$

The radius and the coordinates of the centre of the *release circle* thus must be multiplied by the factor I'_L/I_L or rather E'_L/E_L, where E_L stands for the local phase voltage during an operation of the relay and E'_L the same voltage during a release.

Similarly the radius and the centre coordinates of the *operating circle* must be multiplied by the factor E'_L/E_L in order to account for a varying local voltage. Here E'_L is the local phase voltage at the second operation.

[1] The modified track circuit with the feed impedance placed between the feed transformer and the track will be treated in section VII.

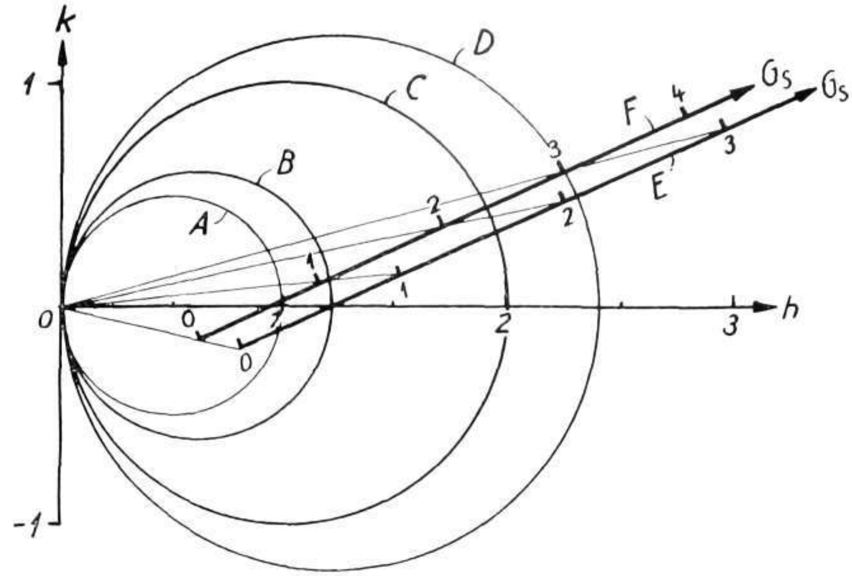


Fig. 11 X 6320

Correction for voltage variations

A not corrected operating circle
B operating circle corrected for a 20 % increase of the local phase voltage
C not corrected release circle
D release circle corrected for a 20 % increase of the local phase voltage
E not corrected shunt line
F shunt line corrected for a 33 % increase of the track feed voltage

In the deduction of the shunt line equation the track feed voltage E was assumed to be constant. If this is not the case the factor E cannot be eliminated when the equations (4) and (5) are divided by equation (6). The right members of the equations (7), (8), (10) and (11) thus will contain the factor E/E' in which E and E' are the track feed voltage values at two different occasions. At the former occasion the track is supposed to be open (*i. e.* $G_s = 0$). At the latter the track is shunted. If E' is larger than E the factor E/E' is less than 1 and then the length of the shunt line as well as its distance from the origin will shrink in the same proportion.

Fig. 11 shows two shunt lines, E and F. One, E, refers to a constant track feed voltage. The other one, F, has been corrected for a 33 % increase of the track feed voltage, *i. e.* the voltage $E' = 1.33\ E$. The length of F is then reduced to 75 % of the length of E and its distance from the origin is also 75 % of that of E. Fig. 11 also contains two release circles C and D and two operating circles A and B. The circles A and C refer to constant local phase voltage but the circles B and D are corrected for a 20 % increase of the local phase voltage. For the release circle this increase is measured between an operation and a release of the relay. For the operating circles the increase is assumed to have taken place between two operations of the relay.

The correction may, however, be simplified as shown in Fig. 12. Instead of one corrected and one original shunt line, only the latter, designated E, is drawn. This line will now also represent the shrunk shunt line in a coordinate system which has been enlarged to correspond to the shrinkage in Fig. 11. The units in this system are marked with underlined figures. In this enlarged system the circles A, B, C and D are drawn. Measured in the units of the original coordinate system the operating circle A and the release circle C are now corrected for an increase of the track feed voltage. The operating circle B and the release circle D have in addition to this been corrected for an increase of the local phase voltage. For the sake of completeness the uncorrected circles G and H are also shown.

This procedure makes for greater clarity, as one and the same shunt line determines the behaviour of the relay for all different voltages represented by the various operate and release circles.

In the fictive example shown in Fig. 12 the voltages are assumed to have been increased. When computing a track circuit it is likewise preferable to make the original circles refer to the lowest voltages. Then a number of corrected circles are drawn as required for determining the release and operating shunt conductances at the normal and the highest voltages.

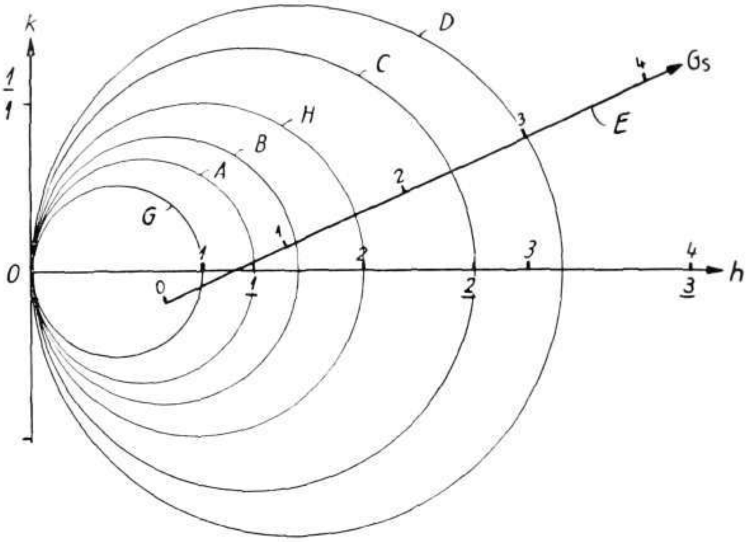


Fig. 12 X 6321

Simplified correction for voltage variations

- A operating circle corrected for a 33 % increase of the track feed voltage
- C corresponding release circle
- B operating circle corrected for a 33 % increase of the track feed voltage and a 20 % increase of the local phase voltage
- D corresponding release circle
- G not corrected operating circle
- H not corrected release circle

VI. *Numerical Example*

Take the case of a track circuit of 2 km length. The frequency is 50 c/s. The rail impedance is $0.64\ \Omega\ |\underline{59^\circ}$ per km. The leakage varies between zero and 1.0 mho per km. The maximum leakage may, however, be either a pure conductance or composed of a pure conductance and a capacitive susceptance with a resulting 45° phase angle. The relay has the following data:

Local phase voltage 220 V
Local phase current 0.212 A
Local phase impedance $1040\ \Omega\ |\underline{74^\circ}$.

Track phase operating voltage 5.6 V
Track phase operating current 0.190 A
Track phase impedance $29.5\ \Omega\ |\underline{72^\circ}$.

$f = 2.0$.

The relay transformer and the track feed transformer have the same values of no-load resp. short-circuit impedances, measured on the track side:

$$Z_{kR} = Z_{kT} = 0.2\ \Omega\ |\underline{12^\circ}$$
$$Z_{oR} = Z_{oT} = 30\ \Omega\ |\underline{55^\circ}$$

The step-up ratio of the relay transformer is 4 : 1.

The feed impedance shall consist of a pure resistance connected to the supply side of the feed transformer and so dimensioned that the combined impedance of the feed transformer and the feed impedance shall be numerically equal to the impedance of the relay transformer with attached relay, both impedances being measured from the track side.

The track feed voltage and the local phase voltage both vary with ± 15 % of the nominal value.

The shunt values by different leakages and varying voltages shall be computed and also the power demand of the track circuit and the step-down ratio required for adapting the track feed transformer to a 220 V supply voltage.

Out of the given values may be computed:

$$Z_R = \frac{1}{4^2}\cdot 29.5\ |\underline{72^\circ} = 1.85\ \Omega\ |\underline{72^\circ} \text{ and } I_{RC_{90^\circ}} = 4\cdot 0.19 = 0.72 \text{ A}$$

$$C_T = C_R = \frac{1}{\sqrt{1 - Y_{oR}\, Z_R}} = \frac{1}{\sqrt{1 - \dfrac{0.2\,\underline{|12^\circ}}{30\,\underline{|55^\circ}}}} \simeq 1$$

$$Z_{Ra} = 1.85\,\underline{|72^\circ} + 0.2\,\underline{|12^\circ} = 1.85\ (\cos\ 72^\circ + j\ \sin\ 72^\circ) +$$
$$+ 0.2\ (\cos\ 12^\circ + j\ \sin\ 12^\circ) = 0.571 + j\,1.76 + 0.195 + j\ 0.04 =$$
$$= 0.766 + j\ 1.8 = 1.95\ \Omega\ \underline{|66.^\circ 9}$$

$$B = 1 + \frac{1.85\,\underline{|72^\circ}}{30\,\underline{|55^\circ}} = 1 + 0.06\,\underline{|17^\circ} \simeq 1.06\,\underline{|1^\circ}$$

The impedance of the relay transformer with attached relay will be

$$\frac{Z_{Ra}}{B} = \frac{1.95\,\underline{|66^\circ.9}}{1.06\,\underline{|1^\circ}} \simeq 1.84\ \Omega\ \underline{|66^\circ}$$

The impedance of the track feed transformer with attached feed impedance is $\dfrac{Z_{Ta}}{A}$ and shall have the same numerical value as $\dfrac{Z_{Ra}}{B}$

A trial computation gives $Z_T = 1.7\ \Omega$ (real)

$$\therefore\ Z_{Ta} = 1.7 + 0.2\ \underline{|12^\circ} \simeq 1.9\,\Omega\ \underline{|1.^\circ 2}$$

$$A = 1 + \frac{1.7}{30\ \underline{|55^\circ}} \simeq 1.04\overline{|3^\circ}$$

$$\therefore\ \frac{Z_{Ta}}{A} = \frac{1.9\ \underline{|1^\circ.2}}{1.04\ \overline{|3^\circ}} = 1.83\ \ \underline{|4^\circ}$$

which well satisfies the desired conditions.

In order to determine the shunt lines the line constants C, Z_k and Y_0 must be computed for a number of varying leakages say 0, 0.5, 1.0, $0.5\,\underline{|45^\circ}$ and $1.0\ \underline{|45^\circ}$ mho/km.

write $\gamma s = \beta s + j\,\alpha s$ then

$$C = \cosh\gamma s = \cosh\beta s \cdot \cos\alpha s + j\sinh\beta s \cdot \sin\alpha s$$
$$\sinh\gamma s = \sinh\beta s \cdot \cos\alpha s + j\cosh\beta s \cdot \sin\alpha s$$
$$\mathrm{tgh}\,\gamma s = \frac{\sinh\ \gamma s}{\cosh\gamma s}$$

The computation is carried out for the case when the leakage equals $g + j\omega c =$ $= 1.0$ mho/km.

$$\gamma s = 2\sqrt{0.64\,\underline{|59^\circ} \cdot 1.0} = 1.6\,\underline{|29^\circ.5} = 1.39 + j\,0.788$$
$$\therefore\ \beta s = 1.39 \text{ and } \alpha s = 0.788 \text{ radians or } 45^\circ.1.$$

A table gives cosh 1.39 = 2.1320 and sinh 1.39 = 1.8829

$\therefore\ \cosh\gamma s = 2.1320 \cdot \cos 45^\circ.1 + j\,1.8829 \cdot \sin 45^\circ.1 =$

(cos $45^\circ.1$ and sin $45^\circ.1$ may be read on a slide rule)

$= 1.505 + j\,1.34 = 2.01\,\underline{|41^\circ.7}$

Now we may calculate $\sinh\gamma s = 2.01\,\underline{|48^\circ.6}$ and

$\mathrm{tgh}\,\gamma s = 1.0\ \ \underline{|6^\circ.9}$

The characteristic impedance of the line

$$Z = \sqrt{\frac{0.64\,\underline{|59^\circ}}{1.0}} = 0.8\ \Omega\ \underline{|29^\circ.5}$$

$\therefore$ The short-circuit impedance of the line

$$Z_k = 0.8\,\underline{|29^\circ.5} \cdot 1.0\,\underline{|6^\circ.9} = 0.8\ \Omega\ \underline{|36^\circ.4} \text{ and}$$

The no-load admittance of the line

$$Y_o = \frac{1.0 \,\underline{|6^\circ.9}}{0.8 \,\underline{|29^\circ.5}} = 1.25\, S\, \overline{|22^\circ.6}$$

The following table gives the computed values of the line constants for the five assumed values of the leakage.

case	$g + j\omega c$ mho/km	Z Ω	C	Z_k Ω	Y_o mho
1	0	∞	1	$1.28\,\underline{|59^\circ}$	0
2	0.5	$1.13\,\underline{|29^\circ.5}$	$1.42\,\underline{|25^\circ.2}$	$1.0\,\underline{|43^\circ.9}$	$0.785\,\underline{|15^\circ}$
3	1.0	$0.8\,\underline{|29^\circ.5}$	$2.01\,\underline{|41^\circ.7}$	$0.8\,\underline{|36^\circ.4}$	$1.25\,\overline{|22^\circ.6}$
4	$0.5\,\underline{|45^\circ}$	$1.13\,\underline{|7^\circ}$	$0.98\,\underline{|36^\circ.7}$	$1.24\,\underline{|34^\circ.3}$	$0.974\,\underline{|20^\circ.3}$
5	$1.0\,\underline{|45^\circ}$	$0.8\,\underline{|7^\circ}$	$1.195\,\underline{|66^\circ.7}$	$1.0\,\underline{|16^\circ.4}$	$1.56\,\underline{|2^\circ.4}$

In order to write the equations (10) and (11) the values of the nominators and denominators are computed according to the following scheme.

case	$B \cdot Z_k$ Ω	$Z_{Ra} + BZ_k$ Ω	$A\,(Z_{Ra} + BZ_k)$ Ω
1	$1.355\,\underline{|60^\circ} = 0.678 + j\,1.175$	$1.444 + j\,2.975 = 3.3\,\underline{|64^\circ.1}$	$3.43\,\underline{|61^\circ.1} = 1.66 + j\,3.0$
2	$1.06\,\underline{|44^\circ.9} = 0.751 + j\,0.748$	$1.517 + j\,2.548 = 2.96\,\underline{|59^\circ.2}$	$3.08\,\underline{|56^\circ.2} = 1.72 + j\,2.56$
3	$0.847\,\underline{|37^\circ.4} = 0.675 + j\,0.515$	$1.441 + j\,2.315 = 2.73\,\underline{|58^\circ.1}$	$2.84\,\underline{|55^\circ.1} = 1.628 + j\,2.33$
4	$1.315\,\underline{|35^\circ.3} = 1.072 + j\,0.76$	$1.838 + j\,2.56 = 3.15\,\underline{|54^\circ.3}$	$3.28\,\underline{|51^\circ.3} = 2.055 + j\,2.56$
5	$1.06\,\underline{|17^\circ.4} = 1.012 + j\,0.394$	$1.778 + j\,2.194 = 2.82\,\underline{|50^\circ.9}$	$2.93\,\underline{|47^\circ.9} = 1.97 + j\,2.175$
case	$Y_o Z_{Ra}$	$B + Y_o Z_{Ra}$	$Z_{Ta}\,(B + Y_o Z_{Ra})$ Ω
1	0	$1.06\,\underline{|1^\circ}$	$2.01\,\underline{|2^\circ.2} = 2.01 + j\,0.077$
2	$1.532\,\underline{|51^\circ.9} = 0.946 + j\,1.208$	$2.01 + j\,1.23 = 2.35\,\underline{|31^\circ.5}$	$4.46\,\underline{|32^\circ.7} = 3.76 + j\,2.42$
3	$2.44\,\underline{|44^\circ.3} = 1.75 + j\,1.705$	$2.81 + j\,1.725 = 3.29\,\underline{|31^\circ.6}$	$6.25\,\underline{|32^\circ.8} = 5.25 + j\,3.39$
4	$1.90\,\underline{|87^\circ.2} = 0.1 + j\,1.90$	$1.16 + j\,1.92 = 2.24\,\underline{|58^\circ.8}$	$4.25\,\underline{|60^\circ.0} = 2.13 + j\,3.68$
5	$3.02\,\underline{|69^\circ.3} = 1.07 + j\,2.83$	$2.13 + j\,2.85 = 3.55\,\underline{|53^\circ.2}$	$6.74\,\underline{|54^\circ.4} = 3.92 + j\,5.48$

case	AZ_k Ω	$Z_{Ta} + AZ_k$ Ω	$Z_{Ra}\,(Z_{Ta} + AZ_k)$ Ω²
1	$1.33\,\underline{|56^\circ.0} = 0.744 + j\,1.102$	$2.644 + j\,1.142 = 2.88\,\underline{|23^\circ.4}$	$5.61\,\underline{|90^\circ.3}$
2	$1.04\,\underline{|40^\circ.9} = 0.786 + j\,0.681$	$2.686 + j\,0.721 = 2.78\,\underline{|15^\circ.0}$	$5.41\,\underline{|81^\circ.9}$
3	$0.832\,\underline{|33^\circ.4} = 0.695 + j\,0.459$	$2.595 + j\,0.499 = 2.64\,\underline{|10^\circ.9}$	$5.15\,\underline{|77^\circ.8}$
4	$1.29\,\underline{|31^\circ.3} = 1.102 + j\,0.67$	$3.002 + j\,0.71 = 3.08\,\underline{|13^\circ.3}$	$6.0\,\underline{|80^\circ.2}$
5	$1.04\,\underline{|13^\circ.4} = 1.012 + j\,0.242$	$2.912 + j\,0.282 = 2.91\,\underline{|5^\circ.5}$	$5.66\,\underline{|72^\circ.4}$
case	$Z_{Ta}\,(Z_{Ra} + BZ_k)$ Ω²	$A\,(Z_{Ra} + BZ_k) + Z_{Ta}\,(B + Y_0 Z_{Ra})$ Ω	
1	$6.26\,\underline{|65^\circ.3}$	$3.67 + j\,3.08 = 4.79\,\underline{|40^\circ.0}$	
2	$5.62\,\underline{|60^\circ.4}$	$5.48 + j\,4.98 = 7.4\,\underline{|42^\circ.3}$	
3	$5.19\,\underline{|59^\circ.3}$	$6.88 + j\,5.72 = 8.95\,\underline{|39^\circ.8}$	
4	$5.98\,\underline{|55^\circ.5}$	$4.19 + j\,6.24 = 7.5\,\underline{|56^\circ.1}$	
5	$5.35\,\underline{|52^\circ.1}$	$5.89 + j\,7.66 = 9.65\,\underline{|52^\circ.5}$	

The leakage in case 3, 1.0 mho/km, is chosen as the basic value. The following shunt line equations may then be developed by inserting the values from the scheme above into the equations (10) and (11).

Case 1:

$$\frac{I_{RC}}{I'_{RD}} = \frac{1}{2.01\,\underline{|41^\circ.7}} \cdot \frac{4.79\,\underline{|40^\circ.0} + G_s \begin{cases} 5.61\,\underline{|90^\circ.3} \\ 6.26\,\underline{|65^\circ.3} \end{cases}}{8.95\,\underline{|39^\circ.8}} =$$

$$= 0.266\,\overline{|41^\circ.5} + G_s \begin{cases} 0.312\,\underline{|8^\circ.8} \\ 0.348\,\overline{|16^\circ.2} \end{cases}$$

The figures to the right of the bracket after G_s refer to the train shunt at the relay end (above) resp. at the feed end (below).

Case 2:

$$\frac{I_{RC}}{I'_{RD}} = \frac{1.42\,\underline{|25^\circ.2}}{2.01\,\underline{|41^\circ.7}} \cdot \frac{7.4\,\underline{|42^\circ.3} + G_s \begin{cases} 5.41\,\underline{|81^\circ.9} \\ 5.62\,\underline{|60^\circ.4} \end{cases}}{8.95\,\underline{|39^\circ.8}} =$$

$$= 0.584\,\overline{|14^\circ.0} + G_s \begin{cases} 0.427\,\underline{|25^\circ.6} \\ 0.444\,\underline{|4^\circ.1} \end{cases}$$

Case 3:

$$\frac{I_{RC}}{I_{RD}} = 1 + \frac{G_s \begin{cases} 5.15\,\underline{|77^\circ.8} \\ 5.19\,\underline{|59^\circ.3} \end{cases}}{8.95\,\underline{|39^\circ.8}} = 1 + G_s \begin{cases} 0.575\,\underline{|38^\circ.0} \\ 0.58\,\underline{|19^\circ.5} \end{cases}$$

Case 4:

$$\frac{I_{RC}}{I'_{RD}} = \frac{0.98\,\underline{|36^\circ.7}}{2.01\,\underline{|41^\circ.7}} \cdot \frac{7.5\,\underline{|56^\circ.1} + G_s \begin{cases} 6.0\,\underline{|80^\circ.2} \\ 5.98\,\underline{|55^\circ.5} \end{cases}}{8.95\,\underline{|39^\circ.8}} =$$

$$= 0.409\,\underline{|11^\circ.3} + G_s \begin{cases} 0.327\,\underline{|35^\circ.4} \\ 0.326\,\underline{|10^\circ.7} \end{cases}$$

Case 5:

$$\frac{I_{RC}}{I'_{RD}} = \frac{1.195\,\underline{|66^\circ.7}}{2.01\,\underline{|41^\circ.7}} \cdot \frac{9.65\,\underline{|52^\circ.5} + G_s \begin{cases} 5.66\,\underline{|72^\circ.4} \\ 5.35\,\underline{|52^\circ.1} \end{cases}}{8.95\,\underline{|39^\circ.8}} =$$

$$= 0.64\,\underline{|37^\circ.7} + G_s \begin{cases} 0.376\,\underline{|57^\circ.6} \\ 0.355\,\underline{|37^\circ.3} \end{cases}$$

The shunt lines are shown in Fig. 13, where they are marked with figures indicating the corresponding leakage case, the letters F (= feed end) and R (= relay end) referring to the location of the train shunt.

It may be observed that the origins of the shunt lines are so situated that they may be circumscribed by an operating circle with diameter = 1 and $\varphi = 90^\circ$. Therefore this value of φ is chosen in the following. See above section IV, Ericsson Review No 2/1947.

In Fig. 13 are shown three operating circles A, B and C and three release circles D, E and H for different voltages.

The circles A and D apply to the lowest voltage values, 85 % of the nominal voltages. The circles B and E apply when the local phase voltage and the

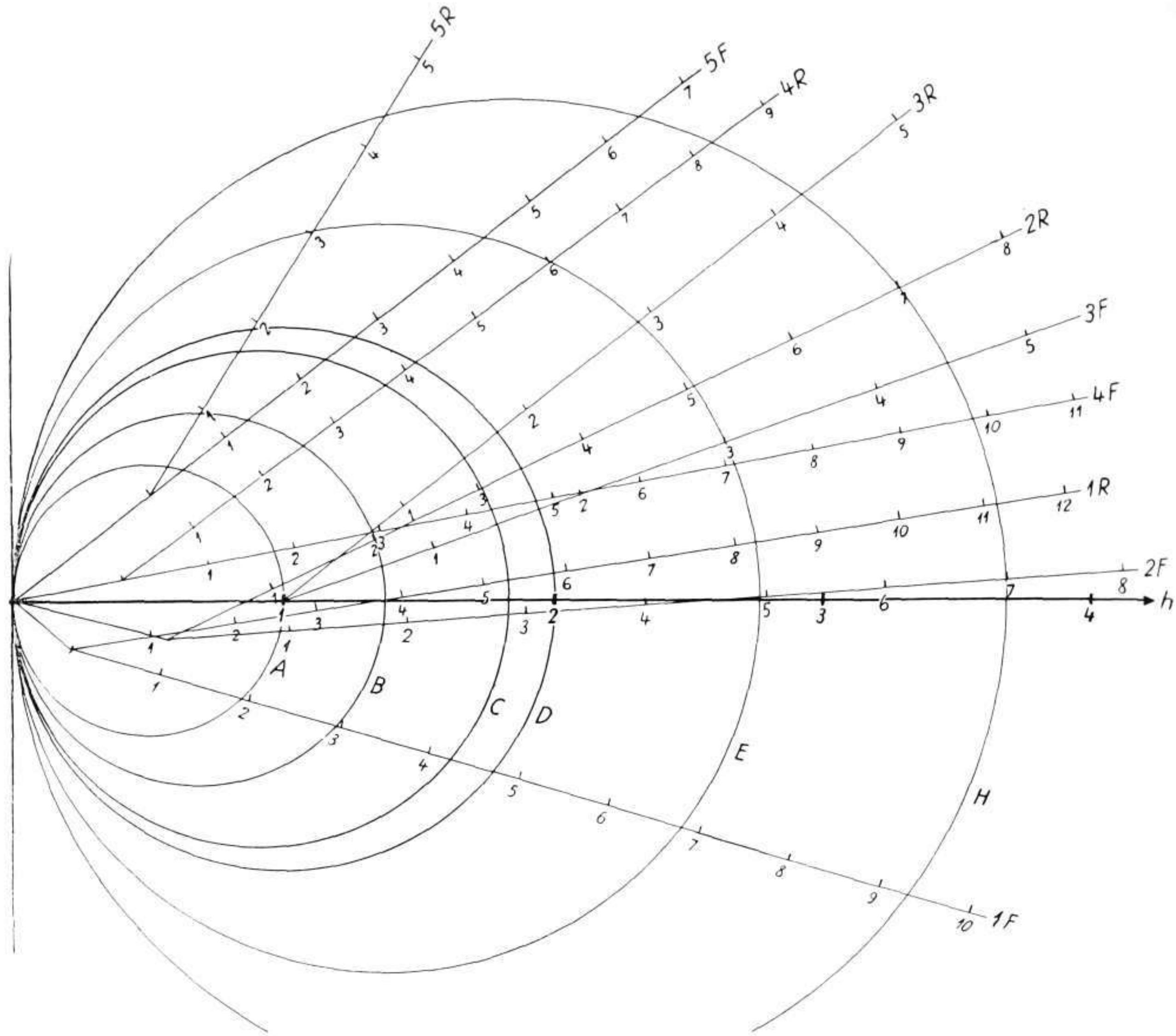


Fig. 13 X 7457

Shunt lines for the feed end (F) and for the relay end (R) at various leakages. Operating and release circles for various increases of voltage.

track feed voltage both have increased from 85 % to 100 % of the nominal values.

The diameters of these circles therefore are $\frac{1}{0.85} \cdot \frac{1}{0.85} = 1.38$ times larger than A resp. D. The circles C and H apply to an increase of voltage from 85 % to 115 % for both current sources. Thus the diameters are enlarged $\left(\frac{1.15}{0.85}\right)^2 = 1.83$ times.

Following shunt conductance values (in mho) are read in Fig. 13.

case	by an voltage increase											
	of 0 %				from 85 % to 100 %				from 85 % to 115 %			
	operating shunt value		release shunt value		operating shunt value		release shunt value		operating shunt value		release shunt value	
	R^*	F^*	R^*	F^*	R^*	F^*	R^*	F^*	R^*	F^*	R^*	F^*
1	2.5	1.9	5.8	4.7	3.8	3.0	8.2	6.8	5.3	4.2	11.1	9.3
2	1.1	0.9	3.3	3.2	2.0	1.8	5.0	5.0	3.0	2.8	7.0	7.0
3	0	0	1.8	1.7	0.7	0.7	2.9	3.0	1.5	1.4	4.2	4.5
4	1.6	1.7	4.1	4.8	2.5	2.9	6.0	7.1	3.7	4.3	8.2	9.8
5	0.3	0.4	1.9	2.7	0.9	1.3	3.0	4.3	1.6	2.3	4.3	6.4

*) R = train shunt at the relay end; F = at the feed end.

The largest release shunt conductance, 11.1 mho, is found by a voltage 15 % above the nominal value in both current sources. At nominal voltage the largest release shunt conductance is 8.2 mho and at 15 % below nominal voltage it is 5.8 mho. This shows the importance of a stable voltage.

Since φ is determined to 90° the required feed voltage E when the track is unoccupied may be computed from ekv. (6). The case 3 gives, since $I_{RC} = 0.72$ A by nominal local phase voltage

$$E = 1 \cdot 2.01 \underline{|41^\circ.7} \cdot 1 \cdot 0.72 \cdot 8.95 \underline{|39^\circ.8} = 13.7 \text{ V} \underline{|81^\circ.5}$$

When the local phase voltage is 85 % of the nominal value I_{RC} must be $\frac{1}{0.85}$ times larger, and E must be increased correspondingly. This will give the minimum value of the feed voltage = 16.1 V. The nominal feed voltage will be $\frac{1}{0.85}$ times the minimum value

or $\left(\frac{1}{0.85}\right)^2 \cdot 13.7 = 19$ V.

The nominal supply voltage = 220 V, thus the step-up ratio of the track feed transformer will be

$$\frac{220}{19} = 11.6$$

The power demand at the largest real leakage (case 3) is

$$P = E \cdot I \cos (\varphi_E - \varphi_I)$$

I may be computed from the equation in the line preceding equ. (5) in section II, by making $G_s = 0$.

$$I = C_T C C_R I_R [B + Y_o Z_{Ra} + Y_{oT} (Z_{Ra} + BZ_k)]$$

The value of the expression inside the bracket is $3.37 \underline{|30^\circ.8}$ and

$I = 1 \cdot 2.01 \underline{|41^\circ.7} \cdot 1 \cdot 072 \cdot 3.37 \underline{|30^\circ.8} = 5.15 \text{ A} \underline{|72^\circ.5}$

at normal local phase voltage. At 15 % lower local phase voltage I will be $\frac{1}{0.85}$ times larger or 6.05 A. The power demand at minimum E and E_L thus will be

$$P = 16.1 \cdot 6.05 \cos (81^\circ.5 - 72.^\circ 5) = 97.5 \cdot \cos 9^\circ = 96.5 \text{ W}$$

At nominal value of E the supplied power is

$$P_{nom} = \frac{P}{(0.85)^2} = 133 \text{ W}$$

From this it may clearly be seen that voltage variations have a very unfavourable influence upon the power demand. For constant voltages it would only amount to 70 W or $(0.85)^4 = 53$ % of the just computed value.

If the local phase is fed from the same supply as the track the local phase current must be given such a phase shift that it lags the desired angle, $\varphi = 90^\circ$, behind the track phase current I_R when the leakage has the chosen basic value of case 3 and the track is unoccupied.Above was computed $E_{nom} = 19.0 \text{ V} \underline{|81^\circ.5}$. Thus the relay current I_R lags 81°.5 behind E. According to the data of the relay, the impedances of the local phase $Z_L = 1040\ \Omega \underline{|74^\circ}$ *i. e.* the local phase current I_L lags 74° behind the local phase voltage. Thus I_R lags $81^\circ.5 - 74^\circ = 7^\circ.5$ behind I_L instead of preceding it 90°. Thus I_L

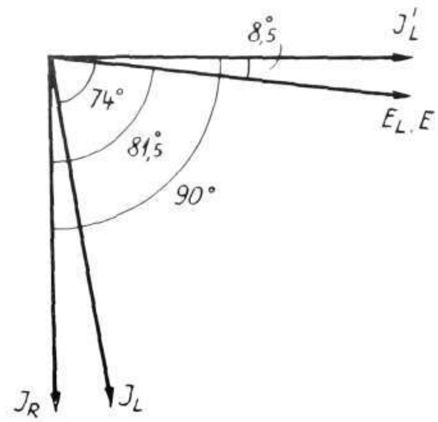


Fig. 14 X 4541

Vector diagram

showing the directions but not the relative size of the e.m.f. of the common current supply E_L, E, the track phase current I_R, the local phase current I_L and the desired local phase current I'_L

must be shifted 97°.5 further backwards until it forms an angle 74° + 97°.5 = = 171°.5 with E_L. This can be done by shifting the local phase terminals and inserting a series condenser with the impedance Z_C in the circuit so that the current precedes E_L an angle 180° — 171°.5 = 8°.5, see Fig. 14. The impedance of the local phase circuit $Z_L + Z_C$ now shall have a phase angle 171°.5 — 180° = —8°.5, see Fig. 15. Out of this figure may be derived

$$Z_C = Z_L \sin 74^\circ + Z_L \cos 74^\circ \operatorname{tg} 8^\circ.5 = 1.0 \cdot Z_L = 1040\ \Omega$$

$$\therefore C = \frac{10^6}{\omega Z_C} = \frac{10^6}{314 \cdot 1040} = 3.06\,\mu F$$

The resulting impedance $Z_L + Z_C = Z_L \cos 74^\circ \dfrac{1}{\cos 8^\circ.5} = 288\ \Omega$ and the voltage $E_L = \dfrac{288}{1040} \cdot 220 = 61$ V. The local phase circuit thus must be connected to the supply voltage through a transformer with the step-down ratio $\dfrac{220}{61} = 3.6$.

VII. Shunt Line Equations for a Modified Arrangement of the Track Circuit

The feed impedance is often inserted between the feed transformer and the track, as in Fig. 16. For this type of track circuit following equations are deduced for U_5, I, U_1 and E. For the other quantities the equations given in section II are unaltered. For the train shunt in the relay end:

$$\begin{aligned} U_5 &= C C_R I_R [Z_{Ra} + B Z_k + Z_T (B + Y_o Z_{Ra}) + G_s Z_{Ra} (Z_T + Z_k)] \\ I &= C_T C C_R I_R [A (B + Y_o Z_{Ra}) + Y_{oT} (Z_{Ra} + B Z_k) + G_s Z_{Ra} (A + Y_{oT} Z_k)] \\ E = U_1 &= C_T C C_R I_R [Z_{Ra} + B Z_k + Z_{Ta} (B + Y_o Z_{Ra}) + G_s Z_{Ra} (Z_{Ta} + Z_k)] \quad (12) \end{aligned}$$

For the train shunt in the feed end:

$$\begin{aligned} U_5 &= C C_R I_R [Z_{Ra} + B Z_k + Z_T (B + Y_o Z_{Ra}) + G_s Z_T (Z_{Ra} + B Z_k)] \\ I &= C_T C C_R I_R [A (B + Y_o Z_{Ra}) + Y_{oT} (Z_{Ra} + B Z_k) + G_s A (Z_{Ra} + B Z_k)] \\ E = U_1 &= C_T C C_R I_R [Z_{Ra} + B Z_k + Z_{Ta} (B + Y_o Z_{Ra}) + G_s Z_{Ta} (Z_{Ra} + B Z_k)] \quad (13) \end{aligned}$$

For the unoccupied track:

$$E = C_T C C_R I_R [Z_{Ra} + B Z_k + Z_{Ta} (B + Y_o Z_{Ra})] \ldots\ldots\ldots \quad (14)$$

The $\begin{Bmatrix}\text{upper}\\\text{lower}\end{Bmatrix}$ expressions in the shunt line equation refer to the $\begin{cases}\text{relay end}\\\text{feed end}\end{cases}$

$$\frac{I_{RC}}{I'_{RD}} = \frac{C'}{C} \frac{Z_{Ra} + B Z'_k + Z_{Ta} (B + Y'_o Z_{Ra}) + G_s \begin{cases} Z_{Ra} (Z_{Ta} + Z'_k) \\ Z_{Ta} (Z_{Ra} + B Z'_k) \end{cases}}{Z_{Ra} + B Z_k + Z_{Ta} (B + Y_o Z_{Ra})} \quad (15)$$

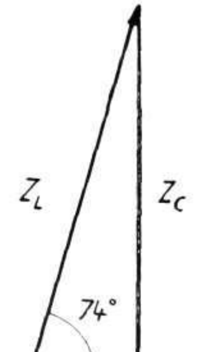


Fig. 15 X 4542

Vector diagram

of the local phase impedance Z_L, the capacitive reactance Z_C and the resulting impedance $Z_L + Z_C$

VIII. The Shunt Line Equation for an Arbitrary Point on the Track

When computing a track circuit it is often necessary to know the required shunt conductance in other points than the terminals. A train must be protected during the whole passage of the track circuit by a released track relay, which lights the stop signal for the next approaching train. In deducing the

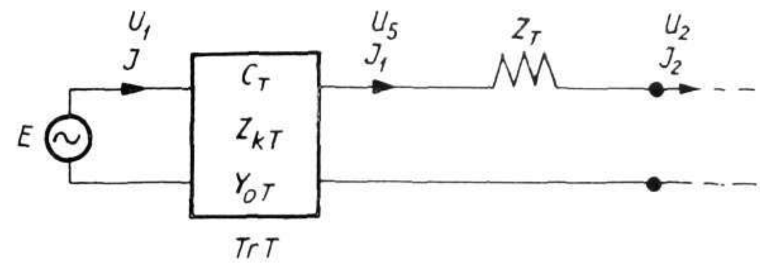


Fig. 16 X 6322

Modified track circuit arrangement with Z_T connected between the track feed transformer and the track

shunt line equation for an arbitrary point of the track we consider the track circuit divided into two parts as in Fig. 17. C_x, Z_{kx}, Y_{ox} and C_y, Z_{ky}, Z_{oy} designate the line constants for the sections x and y respectively. U_x is the track voltage at the shunting point. I_x and I_y are the current values immediately before and after the shunting point.

For U_x and I_y similar equations are valid as for U_2 and I_2 in Fig. 7 with the train shunt in the feed end.

$$U_x = C_y C_R I_R (Z_{Ra} + BZ_{ky})$$
$$I_y = C_y C_R I_R (B + Y_{oy} Z_{Ra})$$

When shunting at a point x km from the feed end, the following equations may be deduced:

$$I_x = I_y + G_s U_x = C_y C_R I_R [B + Y_{oy} Z_{Ra} + G_s (Z_{Ra} + BZ_{ky})]$$

$$U_2 = C_x (U_x + Z_{kx} I_x) = C_x C_y C_R I_R [Z_{Ra} + BZ_{ky} + Z_{kx} (B + Y_{oy} Z_{Ra}) + G_s Z_{kx} (Z_{Ra} + BZ_{ky})]$$

$$I_2 = C_x (I_x + Y_{ox} U_x) = C_x C_y C_R I_R [B + Y_{oy} Z_{Ra} + Y_{ox} (Z_{Ra} + BZ_{ky}) + G_s (Z_{Ra} + BZ_{ky})]$$

$$U_1 = C_T (U_2 + Z_{kT} I_2) = C_T C_x C_y C_R I_R [(Z_{Ra} + BZ_{ky}) (1 + Y_{ox} Z_{kT}) + (B + Y_{oy} Z_{Ra}) (Z_{kx} + Z_{kT}) + G_s (Z_{Ra} + BZ_{ky}) (Z_{kx} + Z_{kT})]$$

$$I = C_T (I_2 + Y_{oT} U_2) = C_T C_x C_y C_R I_R [(Z_{Ra} + BZ_{ky}) (Y_{ox} + Y_{oT}) + (B + Y_{oy} Z_{Ra}) (1 + Y_{oT} Z_{kx}) + G_s (Z_{Ra} + BZ_{ky}) (1 + Y_{oT} Z_{kx})]$$

$$E = U_1 + Z_T I = C_T C_x C_y C_R I_R [(A + Y_{ox} Z_{Ta}) (Z_{Ra} + BZ_{ky}) + (B + Y_{oy} Z_{Ra}) (Z_{Ta} + AZ_{kx}) + G_s (Z_{Ra} + BZ_{ky}) (Z_{Ta} + AZ_{kx})] \quad (16)$$

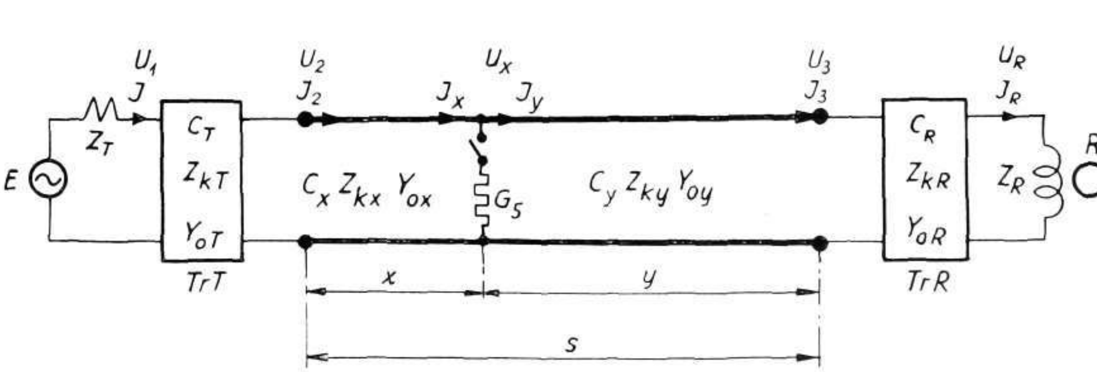
For the unoccupied track we get the connection between E and I_R out of the equation (16) if we put $G_s = 0$

$$E = C_T C_x C_y C_R I_R [(A + Y_{ox} Z_{Ta}) (Z_{Ra} + BZ_{ky}) + (B + Y_{oy} Z_{Ra}) (Z_{Ta} + AZ_{kx})] \ldots\ldots\ldots\ldots \quad (17)$$

Fig. 17 X 7458

Substitute circuit diagram for a track circuit with the train shunt located at the distances x from the feed end and y from the relay end

C_x, Z_{kx}, Y_{ox}	characteristic line constants for the section x
C_y, Z_{ky}, Y_{oy}	characteristic line constants for the section y
U_x	voltage between the rails at the shunting point
I_x, I_y	the current immediately before and after the shunting point



The shunt line equation for a point located x km from the feed end and y km from the relay end is obtained as above in section III. The track phase current I_{RC} for the assumed basic value of the leakage and unoccupied track is divided by the track phase current I'_{RD} for an arbitrary leakage and shunted track.

I_{RC} is obtained from equation (17) and I'_{RD} from equ. (16).

$$\frac{I_{RC}}{I'_{RD}} = \frac{C'_x C'_y}{C_x C_y} \frac{(A + Y'_{ox} Z_{Ta})(Z_{Ra} + BZ'_{ky}) + (B + Y'_{oy} Z_{Ra})(Z_{Ta} + AZ_{kx}) + + G_s (Z_{Ra} + BZ'_{ky})(Z_{Ta} + AZ'_{kx})}{(A + Y_{ox} Z_{Ta})(Z_{Ra} + BZ_{ky}) + (B + Y_{oy} Z_{Ra})(Z_{Ta} + AZ_{kx})} \quad \ldots\ldots\ldots\ldots (18)$$

If the train shunt is located at the feed end $x = 0$; $y = s$; $C_x = 1$; $C_y = C$; $Y_{ox} = Z_{kx} = 0$; $Y_{oy} = Y_o$ and $Z_{ky} = Z_k$.

If these quantities are inserted, equ. (18) will change to equ. (11), as would be expected.

Similarly, if the train shunt is located at the relay end $x = s$; $y = 0$; $C_x = C$; $C_y = 1$; $Y_{ox} = Y_o$; $Z_{kx} = Z_k$; $Y_{oy} = Z_{ky} = 0$ and the equ. (18) then will change to equ. (10).

The equation (17) must be identical with equ. (6), as both give the relation between E and I_R for an unoccupied track. This may be proven as follows:

Insert in equ. (17)

$$C_x = \cosh \gamma x,\ C_y = \cosh \gamma y,\ Y_{ox} = \frac{1}{Z} \operatorname{tgh} \gamma x,\ Y_{oy} = \frac{1}{Z} \operatorname{tgh} \gamma y,$$

$$Z_{kx} = Z \operatorname{tgh} \gamma x \text{ and } Z_{ky} = Z \operatorname{tgh} \gamma y.$$

$$\therefore E = C_T C_R I_R \left[\left(A \cosh \gamma x + \frac{Z_{Ta}}{Z} \sinh \gamma x \right)(Z_{Ra} \cosh \gamma y + BZ \sinh \gamma y) + \right.$$

$$\left. + \left(B \cosh \gamma y + \frac{Z_{Ra}}{Z} \sinh \gamma y \right)(Z_{Ta} \cosh \gamma x + AZ \sinh \gamma x) \right] =$$

$$= C_T C_R I_R \left[(AZ_{Ra} + BZ_{Ta})(\cosh \gamma x \cosh \gamma y + \sinh \gamma x \sinh \gamma y) + \right.$$

$$\left. + \left(ABZ + \frac{Z_{Ra} Z_{Ta}}{Z} \right)(\cosh \gamma x \sinh \gamma y + \sinh \gamma x \cosh \gamma y) \right] =$$

$$= C_T C_R I_R \left[(AZ_{Ra} + BZ_{Ta}) \cosh \{\gamma (x + y)\} + \right.$$

$$\left. + \left(ABZ + \frac{Z_{Ra} Z_{Ta}}{Z} \right) \sinh \{\gamma (x + y)\} \right] =$$

$$= C_T C_R I_R \cosh \gamma s [AZ_{Ra} + BZ_{Ta} + ABZ_k + Z_{Ra} Z_{Ta} Y_o] =$$

$$= C_T C C_R I_R [A(Z_{Ra} + BZ_k) + Z_{Ta}(B + Y_o Z_{Ra})]$$

i. e. equ. (6)

As the right member of equ. (17) forms part of the right member of equ. (16) it follows that equ. (16) may be written:

$$E = C_T C_R I_R [C \{A(Z_{Ra} + BZ_k) + Z_{Ta}(B + Y_o Z_{Ra})\} + + G_s C_x C_y (Z_{Ra} + BZ_{ky})(Z_{Ta} + AZ_{kx})] \quad \ldots\ldots (16\,a)$$

Then equ. (18) takes the following form:

$$\frac{I_{RC}}{I'_{RD}} = \frac{C' \{A(Z_{Ra} + BZ'_k) + Z_{Ta}(B + Y_o Z_{Ra})\} + G_s C'_x C'_y (Z_{Ra} + BZ'_{ky})(Z_{Ta} + AZ'_{kx})}{C[A(Z_{Ra} + BZ_k) + Z_{Ta}(B + Y_o Z_{Ra})]} \quad \ldots\ldots (18\,a)$$

Equ. (18 a) resembles the equations (10) and (11) except for the factor for G_s which differs in all three equations.

The *centre* of the track length constitutes a special case. Here we have:

$$x = y = \frac{s}{2},\ C'_x = C'_y = \cosh \frac{\gamma'_s}{2} = \sqrt{\frac{\cosh \gamma'_s + 1}{2}} = \sqrt{\frac{C' + 1}{2}}$$

$$Y'_{0x} = Y'_{0y} = \frac{1}{Z'} \operatorname{tgh} \frac{\gamma'_s}{2} = \frac{1}{Z'} \sqrt{\frac{\cosh \gamma' s - 1}{\cosh \gamma' s + 1}} = \frac{1}{Z'} \sqrt{\frac{C' - 1}{C' + 1}} = \frac{C'}{C'} \sqrt{\frac{C' + 1}{C' + 1}} \cdot$$

$$\cdot \frac{1}{Z'} \cdot \sqrt{\frac{C' - 1}{C' + 1}} = \frac{C'}{C' + 1} \cdot \frac{1}{Z'} \sqrt{\frac{(C')^2 - 1}{C'}} = \frac{C'}{C' + 1} \cdot \frac{1}{Z'} \sqrt{\frac{\cosh^2 \gamma' s - 1}{\cosh \gamma' s}} =$$

$$= \frac{\overset{+}{(-)}\ C'}{C' + 1} \cdot \frac{1}{Z'} \operatorname{tgh} \gamma' s = \frac{C'}{C' + 1} \cdot Y_0$$

Similarly may be shown that

$$Z'_{kx} = Z_{ky} = \frac{C'}{C' + 1} \cdot Z'_k$$

If these expressions are inserted in equ. (18 a) we obtain the shunt line equation for the centre of the track length.

$$\frac{I_{RC}}{I'_{RD}} = \frac{C'}{C} \cdot \frac{A\,(Z_{Ra} + BZ'_k) + Z_{Ta}\,(B + Y'_0 Z_{Ra}) + G_s \cdot \frac{C' + 1}{2\,C'} \left(Z_{Ra} + \frac{C'}{C' + 1} \cdot BZ'_k\right) \cdot \left(Z_{Ta} + \frac{C'}{C' + 1} \cdot AZ'_k\right)}{A\,(Z_{Ra} + BZ_k) + Z_{Ta}\,(B + Y_0 Z_{Ra})} \qquad (19)$$

So far shunt line equations have been deduced for track circuits built according to the diagrams in Figs. 1 and 16. Using the basic equations (3) in section II shunt line equations for other types of track circuits may be similarly deduced. If the relay transformer is omitted, all formulas relating to Fig. 1 are valid if we write $Z_{Ra} = Z_R$ and $B = 1$.

IX. The Application of the Shunt Line Equation for Analysis of the Shunt Value for the Track Circuit in Fig. 1

The shunt line equation provides a means of analytical treatment of several problems such as the variation of the shunt value along the length of the track, its dependence of the track length, its relation to the values of Z_T and Z_R and so on. We will now treat a few of these problems but it must be pointed out that hereby the possibilities of the shunt line equation are far from exhausted.

The variation of the shunt value along a symmetrical track circuit

a) From the shunt line equations (10) and (11) it follows that if

$$\frac{Z_{Ta}}{A} = \frac{Z_{Ra}}{B}$$

the factors for G_s will be equal in both equations.

These equations will be identical, *i. e.* the shunt values for the relay end and for the feed end will be equal. $\frac{Z_{Ra}}{B}$ is the impedance of the relay transformer with attached relay measured from the track side, and $\frac{Z_{Ta}}{A}$ is the corresponding impedance of the track feed transformer. If these impedances are equal, the track circuit is symmetrical and it is then self-evident that the shunt values for both ends are equal. From the symmetry it also follows, that the shunt values for point equally distant from the centre of the track also will be equal.

b) The shunt line equation (18 a) applying to an arbitrary point situated on the distances x and y from the feed resp. relay ends contains quantities dependent on x and y only in the factor for G_s. Thus an investigation of the variation of the shunt value with the position of the shunting point may be restricted to the factor for G_s. This factor we designate F.

$$F = C'_x C'_y (Z_{Ra} + BZ'_{ky})(Z_{Ta} + AZ'_{kx}) =$$

$$= C'_x C'_y A B \left(\frac{Z_{Ra}}{B} + Z'_{ky}\right)\left(\frac{Z_{Ta}}{A} + Z'_{kx}\right)$$

When Z_R and Z_T are matched to the characteristic impedance Z' of the line at the time when the shunting occurs, *i. e.* if

$$\frac{Z_{Ra}}{B} = \frac{Z_{Ta}}{A} = Z'$$

then $F = C'_x C'_y A B (Z' + Z'_{ky})(Z' + Z'_{kx}) =$

$$= A B (Z')^2 \cosh \gamma' x \cosh \gamma' y (1 + \operatorname{tgh} \gamma' y)(1 + \operatorname{tgh} \gamma' x) =$$
$$= A B (Z')^2 (\cosh \gamma' y + \sinh \gamma' y)(\cosh \gamma' x + \sinh \gamma' x) =$$
$$= A B (Z')^2 [\cosh \{\gamma' (x + y)\} + \sinh \{\gamma' (x + y)\}] =$$
$$= A B (Z')^2 C' (1 + \operatorname{tgh} \gamma' s)$$

thus independent of x and y.

If the feed impedance element and the relay impedance could be matched to the line characteristic at the shunting moment, the shunt value would be constant along the whole length of the track as long as the leakage does not change.[1] This shunt value, however, is dependent on the leakage so that under certain conditions the shunt conductance as a rule increases as the leakage decreases. This is the case in the example above. (The conditions we allude to imply that the operating and release circles are so chosen that φ nearly equals 90°. If in the example above $\varphi > 90°$ it would well be possible to draw a release circle so that, at least for real leakages, the circle cuts the shunt lines *1F* and *1R* at smaller shunt conductance values than the corresponding values on the shunt lines *3F* and *3R*. In this case the shunt conductance would decrease when the leakage decreases.) Actually, however, such a matching to the line impedance cannot be permanently done, as it would require a frequent shifting of the step-up ratios of the transformers.

c) From the shunt line equations (18) following interesting conclusion can be drawn: If the leakage by some means (for instance by inserting adjustable resistances between the rails) is held constant and if the relay impedance Z_R and the track feed impedance Z_T are matched to the characteristic line impedance for this constant leakage then the shunt value is not only constant along the whole length of the track but also independent of the length of the track. Thus the track circuit may be given any length, even infinite.

The proof is as follows: Constant leakage implies that all quantities in the equ. (18) which are marked with a prime sign are identical with corresponding quantities without this sign. Thus the equation (18) for constant leakages transforms into:

$$\frac{I_{RC}}{I_{RD}} = 1 + G_s \cdot \frac{(Z_{Ra} + BZ_{ky})(Z_{Ta} + AZ_{kx})}{(A + Y_{ox} Z_{Ta})(Z_{Ra} + BZ_{ky}) + (B + Y_{oy} Z_{Ra})(Z_{Ta} + AZ_{kx})}$$

Insert $\dfrac{Z_{Ra}}{B} = \dfrac{Z_{Ta}}{A} = Z$

$$\frac{I_{RC}}{I_{RD}} = 1 + G_s \frac{AB (Z + Z \operatorname{tgh} \gamma y)(Z + Z \operatorname{tgh} \gamma x)}{AB [(1 + \operatorname{tgh} \gamma x)(Z + Z \operatorname{tgh} \gamma y) + (1 + \operatorname{tgh} \gamma y)(Z + Z \operatorname{tgh} \gamma x)]} =$$

$$= 1 + G_s \frac{Z}{2}$$

i. e. independent of s.

[1] Deduced in a different way by *Gall*. See footnote page 38 in Ericsson Review No 2/1947.

The use of adjustable resistances for keeping the leakage constant is not so simple in practice. Probably one has to be contented with a compromise, using fixed resistances which limit the minimum leakage but, unfortunately, at the same time increasing the maximum leakage.

The question of resorting to such means of keeping the shunt value constant or nearly constant along the track may arise in the case when light vehicles with only a few shafts traffic the track. Braking combined with sanding of the track may imperil the contact between the wheels and the rails so that the relay will operate. In such a case and if the shunt conductance value is higher along the track than at ends, it may happen that the vehicle after passing the sanded part of the rail still would fail to release the relay until arriving close to the end of the track circuit. If, on the other hand, the shunt conductance value is nearly constant along the track the relay would be released again, as soon as the vehicle has passed the sanded portion.

For long trains with many shafts it is very improbable that all wheels would be simultaneously insultated from the rails. Then the question of the variation of the shunt value along the length of the track would be less important.

The possibility of building very long track circuits may bring economic gains on railways with large block distances. A condition for the solution of this problem is, as above is said, that the leakage between the rails can be held reasonably constant.

d) How does the shunt conductance value vary along the track circuit when Z_R and Z_T are not matched to the prevailing characteristic line impedance? As above under b), only the factor for G_s has to be examined.

Write:

$$\frac{Z_{Ra}}{B} = pZ' \text{ and } \frac{Z_{Ta}}{A} = qZ'$$

where p and q may be vectors. The factor for G_s

$$F = C_x' C_y' (Z_{Ra} + BZ_{ky}') (Z_{Ta} + AZ_{kx}') \text{ will then be}$$

$$F = C_x' C_y A B (Z')^2 (p + \operatorname{tgh} \gamma' y) (q + \operatorname{tgh} \gamma' x) =$$

$$= A B (Z')^2 (p \cosh \gamma' y + \sinh \gamma' y) (q \cosh \gamma' x + \sinh \gamma' x)$$

1) If $p = q = 1$, *i. e.* when the end impedances are matched to the line

$$F = AB (Z')^2 (\cosh \gamma' s + \sinh \gamma' s)$$

thus independent of x and y, as under b)

2) If $p = q << 1$

$$F \cong A B (Z')^2 \sinh \gamma' x \sinh \gamma' y$$

but $\sinh \varphi \sinh \psi = \frac{1}{2} \cosh (\varphi + \psi) - \frac{1}{2} \cosh (\varphi - \psi)$

$$\therefore F \cong A B (Z')^2 \left[\frac{1}{2} \cosh \gamma' s - \frac{1}{2} \cosh \gamma' (x - y) \right]$$

but $x - y = x - (s - x) = 2x - s$

$$\therefore F \cong \frac{1}{2} A B (Z')^2 [\cosh \gamma' s - \cosh \gamma' (2x - s)]$$

For $x = 0$, $F = 0$

For $x = \frac{s}{2}$, $F \cong \frac{1}{2} A B (Z')^2 (\cosh \gamma' s - 1)$

For $x = s$, $F = 0$

Thus with increasing x, F will grow from zero to a largest vectorial value for $x = \frac{s}{2}$ and will then shrink to zero for $x = s$.

If a shunt line is drawn for every x-value, the scale divisions on these shunt lines will be smallest, approaching zero, for the ends and largest for $x = \frac{s}{2}$. The slope of the shunt lines will also vary. The intersections between the shunt lines and a release circle determine the required shunt values. As the release circle may be drawn in an infinite number of ways, nothing very definite can be stated of the variation of the shunt conductance value as x varies, except that this variation is symmetrical about the centre $\left(x = \frac{s}{2}\right)$ and that it approaches infinity at the ends ($x = 0$; $x = s$).

Generally speaking, one may say that if Z_T and Z_R are smaller than Z' the shunt conductance is larger at the ends than at points nearer the centre of the track.

3) If $p = q >> 1$

$$F \cong A B (Z')^2 pq \cosh \gamma' x \cosh \gamma' y$$

but $\cosh \varphi \cosh \psi = \frac{1}{2} \cosh (\varphi + \psi) + \frac{1}{2} \cosh (\varphi - \psi)$

$$\therefore F \cong \frac{1}{2} A B (Z')^2 pq [\cosh \gamma' s + \cosh \gamma' (2x - s)]$$

For $x = 0$, $F \cong A B (Z')^2 pq \cosh \gamma' s$

For $x = \frac{s}{2}$, $F \cong A B (Z')^2 pq \left(\frac{\cosh \gamma' s}{2} + \frac{1}{2}\right)$

For $x = s$, $F \cong A B (Z')^2 pq \cosh \gamma' s$

When x increases from zero, F will decrease until $x = \frac{s}{2}$ and will then increase again. The length of the divisions of the shunt lines thus will be smaller as the train shunt approaches the middle point and the slope will vary at the same time.

For the same reasons as above nothing very definite can be said about the variation of G_s as x varies, but in practice the required shunt conductance value generally will be smaller at the ends than along the track.

Summing up, we may state the following as regards the practical cases:

High end impedances require more effective shunting along the track than at the ends, low end impedances require more effective shunting at the ends than along the track and end impedances matched to the characteristic line impedance require the same shunt value along the whole length of the track.

X. Low- or high-ohmic Relay?

For a normal track circuit, where no adjustment is done for leakage variations (as under IX b)) and where these are not counteracted (as under IX c)) the variation of the shunt conductance value along the track will depend on the leakage, as the relay and feed impedances, which are constant, will equal the characteristic line impedance only at a certain leakage value. Assume that the end impedances are matched to the line at a mean value of the leakage. When the leakage is large, the characteristic line impedance is low, and then the end impedances will be higher than the characteristic line impedance. When the leakage is small, the opposite will be the case. Such a track circuit would not be very suitable, because when the leakage is large it will be more difficult to release the relay along the track than at the ends. (See under IX d)). For practical reasons a measurement of the shunt value is done at the ends, and such a measurement will then give a false impression of the safe functioning of the relay.

In order to assure as safe a release of the relay in the middle of the track as at the ends, *the relay and the feed impedance ought to be matched to the characteristic line impedance at the highest occuring leakage. Thus the relay and feed impedance element ought to be low-ohmic, i. e.* the relay transformer, if such a one is used, should transform the relay impedance to a low value. The same applies to the feed transformer[1].

Against this may be argued that the shunt conductance value will be larger for low than for high impedances, *i. e.* that it will be easier to release the relay in the latter case. It must be observed, however, that this chiefly holds true for shunting at the ends but not along the track. For long track circuits and large leakages the shunt value along the track will even be independent of the relay and track feed impedances but will be determined by the characteristic line impedance. If we presume that the leakage equals the reference value, the shunt line equation (18 a) for an arbitrary point will take the form

$$\frac{I_{RC}}{I_{RD}} = 1 + G_s \frac{(Z_{Ra} + BZ_{ky})\,(Z_{Ta} + AZ_{kx})}{(A + Y_{0x}Z_{Ta})\,(Z_{Ra} + BZ_{ky}) + (B + Y_{0y}Z_{Ra})\,(Z_{Ta} + AZ_{kx})} =$$

$$= 1 + G_s \cdot \frac{1}{\dfrac{A + \dfrac{Z_{Ta}}{Z}\,\text{tgh}\,\gamma x}{Z_{Ta} + AZ\,\text{tgh}\,\gamma x} + \dfrac{B + \dfrac{Z_{Ra}}{Z}\,\text{tgh}\,\gamma y}{Z_{Ra} + BZ\,\text{tgh}\,\gamma y}} =$$

$$= 1 + G_s \cdot Z \cdot \frac{1}{\dfrac{AZ + Z_{Ta}\,\text{tgh}\,\gamma x}{AZ\,\text{tgh}\,\gamma x + Z_{Ta}} + \dfrac{BZ + Z_{Ra}\,\text{tgh}\,\gamma y}{BZ\,\text{tgh}\,\gamma y + Z_{Ra}}}$$

For increasing values of x and y the terminals of the vectors $\text{tgh}\gamma x$ and $\text{tgh}\gamma y$ will describe a spiral line approaching the value 1. (In the example above, in section VI we had for a length of 2 km a value $\text{tgh}\gamma y = 1.0 \underline{|6°.9}$.) This value gives

$$\frac{I_{RC}}{I_{RD}} = 1 + G_s \cdot Z \cdot \frac{1}{\dfrac{AZ + Z_{Ta}}{AZ + Z_{Ta}} + \dfrac{BZ + Z_{Ra}}{BZ + Z_{Ra}}} = 1 + G_s \cdot \frac{Z}{2}$$

thus independent of Z_{Ra} and Z_{Ta}.

In one end of the track, for instance the relay end, where $y = 0$ and $x = s$, the shunt line equation will take the form:

$$\frac{I_{RC}}{I_{RD}} = 1 + G_s \cdot Z \cdot \frac{1}{\dfrac{AZ + Z_{Ta}\,\text{tgh}\,\gamma s}{AZ\,\text{tgh}\,\gamma s + Z_{Ta}} + \dfrac{BZ}{Z_{Ra}}}$$

which for $\text{tgh}\,\gamma s = 1$ transforms into

$$\frac{I_{RC}}{I_{RD}} = 1 + G_s \cdot Z \cdot \frac{1}{1 + \dfrac{BZ}{Z_{Ra}}}$$

[1] This rule has not been obeyed in the premises for the numerial example under VI.

The factor for G_s increases with increasing Z_{Ra} and finally approaches Z. Thus an increase of the relay impedance will only benefit the relay end.

The same can be proved for the feed end.

For track circuits of shorter length than those now investigated or for the case that the leakage is varying, the matters are not quite so simple to account for mathematically. Z_{Ra} and Z_{Ta} then will not disappear from the shunt line equation, and the required shunt value along the track will depend on these impedances. The tendency of the characteristic line impedance to dominate in the expression for the shunt value remains, however, and will be more pronounced the larger the leakage or the longer the track circuit is.

B. Alternating Current Track Circuits with Single-phase Relays

In the single-phase (single element) relay only one magnet flux is produced. This flux is then split into two parts, differing as to time and space. These part fluxes induce currents in a movable conductor and the action of the fluxes on these currents will cause the conductor to move. So far there is a similarity to the two phase (two-element) relay, but the relay is none the less a single-phase relay, as both part fluxes are produced by *one* and the same winding. A two-phase relay may be connected so as to work as a single-phase relay, for instance by connecting a condenser in series with one phase winding and then connecting this series circuit parallel with the other phase winding to two terminals.

Characteristic for the single-phase relay and the two-phase relay wired as a single-phase relay is that the operating current is single-valued as is also the release current. Thus the ratio between these currents is single-valued. The operating and release circles are concentric with their centres in the origin as already was mentioned, see Fig. 5. The lack of frequency selectivity limits the use of the single-phase relay to circuits where no disturbing currents appear. Thus as a track relay it may be employed *only* at *not* electrified railroads and where the danger of stray alternating currents is non-existent.

As the shunt line equations given above under A are independent of the relay type, they may be applied unaltered to single-phase track circuits. The computation of the shunt conductance value is done as under A by finding the intersection between the shunt line and the respective circle.

Voltage variations affect the circle diagram only to that extent, that correction has to be made for variations in the track feed voltage E. Thus the voltage variations do not influence the shunt values and the power demand as much as in two-phase track circuits, where also the variations in the local phase voltage influence these quantities.

C. Direct Current Track Circuits

Here matters will be still more simplified, as all phase angles disappear, *i. e.* all currents and voltages are in phase and all impedances and admittances turn into resistances and conductances. As no transformers appear, the ratios A and B will equal 1. Z_{Ra} will be exchanged by the relay resistance R_R and Z_{Ra} by the track feed resistance R_T.

The shunt lines will run along the real axis and thus there is no need for drawing a diagram. The ratio between I_{RC} and I'_{RD} is real and equals f. Thus the shunt conductance will be calculated directly from the shunt line equations given under A with the above mentioned simplifications inserted.

Direct current track circuits have earlier been computated by other authors and thus there is no need of further treatment of this subject here.